Bond Finance, Bank Finance, and Bank Regulation

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Abstract

The analysis of optimal bank regulation in a general equilibrium setting could be misleading if the framework does not include the bond market. In this paper, we build a continuous-time macro-finance model in which firms could use both bond-financing and bank-financing. Risky firms appreciate bank credit because banks are efficient at liquidating assets for troubled firms. However, risky firms must pay the risk premium for banks' exposure to aggregate risks. With our framework, we show that if an economy relies more on bond-finance because its bond market is more developed, its optimal capital requirement ought to be more lenient. By contrast, if bond credit is more prominent in an economy because it has more safe firms that mainly use bond-finance, then the optimal capital requirement should be tighter in this economy.

Keywords: bank credit, bond credit, capital requirement, and macro-prudential regulation

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Introduction

To investigate the general equilibrium impact and the macro-prudential role of bank regulation, we cannot ignore the bond market since firms, at least some of them, can resort to bond issuance if bank regulation depresses the supply of bank credit. More importantly, the discussion of optimal bank regulation could be misleading if we overlook the firm-level heterogeneity and the country-level heterogeneity with respect to the relative importance of the bond market. At the firm level, large firms with high credit ratings can easily access the bond market and small and medium-sized enterprises primarily rely on bank-finance. At the country level, in economies like U.S., the bond market is comparable with the bank loan market. But, for countries like China and Japan, bank credit is still the most important external credit that firms rely on. In this paper, we show that such heterogeneity actually can lead to completely opposite policy prescriptions.

This paper proposes a continuous-time macro-finance framework with a productive expert sector, a less productive household sector, and an explicit banking sector. In the production sector, there are safe firms and risky firms. Both types of firms can access the bond market and the loan market. The difference between bond-finance and bank-finance is that banks are more efficient in terms of liquidating troubled firms' assets (Bolton and Freixas, 2000). The net interest spread charged by banks compensate their exposure to aggregate risk that they take via their lending. Households can hold corporate bonds directly or deposit their savings into banks.

The key result of our paper is that the optimal bank regulation could be completely different for two seemly identical economies. In particular, we show that if an economy relies more on bank-finance because it has a less developed bond market, then the optimal level of bank regulation for this economy ought to be more stringent in the economy. However, if the loan market is more important in an economy because there are many risky firms that mainly use bank loans, the optimal bank regulation in this economy should be relatively lenient.

In addition to the theoretical contributions, our model captures two empirical facts of bond-financing and bank-financing. The first fact is that bank-financing tend to be more volatile and cyclical than bond-financing *in the long run* (Becker and Ivashina, 2014). During *financial crises*, however, we observe that firms, especially those with relatively high credit ratings, tend to substitute bank credit with bond credit when the supply of bank loans shrinks substantially (Adrian et al., 2012). Next, we illustrate the main mechanisms of our model and outline intuitions for main contributions of our paper.

In our framework, risky firms tend to prefer bank credit and safe ones mainly rely on bond credit. Since banks can liquidate troubled firms' asset in a more efficient way, they request less compensation for bankruptcy costs than bondholder do. The liquidation efficiency of bank credit is more important for risky firms than for safe firms because the likelihood that safe firms would have to face costly liquidation is tiny. This is consistent with empirical findings in Rauh and Sufi (2010) and Becker and Josephson (2016). Bank credit does not always dominate bond credit for risky firms. This is because risky firms must pay banks the risk premium for the aggregate risk that banks are exposed to. Hence, when the net interest spread that banks charge rises, risky firms will replace bank-finance with bond-finance.

The net interest spread depends on the financial health of the intermediary sector. When the banking sector is well capitalized, it channels a large amount of funds from households to firms, which leads to the high supply of bank loans. Since the leverage that banks take is relatively low, their exposure to aggregate risk is low as well. Hence, bank credit is relatively cheap when the banking sector has adequate equity capital. In addition to the leverage of the banking sector, the net interest spread is also a function of aggregate risk, which endogenously fluctuates over business cycle.

Although we assume that the size of aggregate shocks is constant over time, their endogenous effects on the economy varies because the effects of financial amplification depend on balance sheets of both banks and experts (Bernanke et al., 1999; Kiyotaki and Moore, 1997). Suppose a series of adverse shocks hit the economy, both bank capital and productive experts' net worth decline disproportionately due to their use of leverage. As a result, the supply of bank loans shrinks, which in turn lowers experts' holdings of assets and the aggregate productivity as well as asset prices. The depreciation of asset prices hurts balance sheets of both banks and experts, which in turn lowers the supply of bank loans and experts' holdings of assets further. We name the effect of the financial amplification as endogenous risk.

Bank-finance is pro-cyclical in our model. During economic upturns when the financial health of the banking sector improves, it is relatively cheap to raise bank credit. Thus, more firms choose bank-financing, and these firms also take high leverage because of low endogenous risks.

Bond-financing is less volatile than bank-financing as a result of two opposing effects. At the extensive margin, less firms choose bond credit during upturns when bank loans are relatively cheap. However, at the intensive margin, firms that still use bond-finance would like to issue more bonds because endogenous risk is low during upturns. Thus, the total bond credit does not vary much in upturns. In financial crises, more firms issue bonds because bank loans become more expensive. Moreover, in the presence of low asset prices, firms tend to take high leverage since the returns of holding assets are high. Therefore, the rise in bond credit in crises can to some extent make up the loss caused by the decline in the supply of bank loans.

Bank regulation in our framework could improve social welfare because there exist pecuniary externalities that experts and bankers do not internalize the impact of their leverage decisions on endogenous risk (Lorenzoni, 2008; Stein, 2012). Hence, bank regulation can adjust bankers' leverage and the supply of bank loans, which in turn may lower endogenous risks and improve social welfare. In this paper, we focus on the bank regulation that banks are required to main a minimum capital ratio, which is widely implemented in most economies.

Based on our framework, we highlight that the discussion of optimal capital ratio requirement could be misleading if we do not examine the underlying reason why bond-finance is relatively more or less prominent in an economy. In particular, we have conducted three types of comparative statics analyses. Firstly, we adjust the efficiency of asset liquidation by bondholders. In economies with a more advanced bond market, the cost is lower if bondholders liquidate a firm's assets. Secondly, we vary the fraction of risky firms in an economy. Lastly, we fix the fraction of risky firms and change how likely risky firms default in an economy.

In the first type of comparative statics analysis, we find that the optimal level of capital requirement should be *more lenient* in an economy with a better bond market, in which *bank credit is less prominent*. In an economy with a more advanced bond market, if capital requirement tightens, risky firms tend to easily substitute bank credit with bond credit. Therefore, tightening capital requirement does not substantially lower the overall leverage of the production sector. Moreover, given that the capital requirement is sufficiently stringent and a large number of many risky firms start using bond-finance, the financial stability of the economy actually deteriorates as the required capital ratio increases. This is because if risky firms use more bond credit, their marginal borrowing costs would increase, which implies that the decline in asset prices in downturn has to be large enough so that risky firms would like to hold more assets. Therefore, the asset fire-sale is more significant when risky firms issue more corporate bonds.

Our second thought experiment shows that the optimal capital requirement ought to be *more lenient* in an economy with more risky firms, in which *bank credit is more prominent*. This result contrasts with the previous one because the similar policy prescription is offered to two economies that have completely opposite profiles of their financial markets. The intuition for the second result is straightforward. If there are more firms that rely on bank credit, tightening capital requirement could lower the external credit that risky firms can raise and thus decrease the average productivity of the economy, which in turn would make social welfare worse off.

In the third experiment, we find that if the risky firms in an economy become less risky, that is, *bank credit becomes less prominent*, the optimal capital requirement should become *more lenient*. The intuition is similar to that for the first thought experiment. If risky firms becomes less risky, tightening bank regulation will force risky firms to switch to more costly bond credit, which in turn could lower the financial stability of the economy.

Related Literature. Our paper is related to four strands of literature. First of all, our paper uses a continuous-time macro-finance framework that emphasizes the financial amplification mechanism (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2012; Di Tella, 2012). The major contribution of our paper is that we explicitly model a financial intermediary sector rather than grouping the real sector and financial intermediary sector together. With the new framework that we propose, we can explicitly analyze the macroeconomic implications of bank regulation. In the new framework, we can also see that there are two layers of financial amplification, one at the firm-level and the other one at the intermediary-level.

Secondly, since the 2007-2009 financial crisis, there have been a fast-growing number of papers that investigate the macro-prudential role of bank regulation in a dynamic general equilibrium framework (Van den Heuvel, 2008; Christiano and Ikeda, 2013; Martinez-Miera and Suarez, 2014; Derviz et al., 2015; Begenau, 2016; Elenev et al., 2016). This literature largely builds on the traditional research on bank regulation, which typically apply static and mostly partial equilibrium models to investigate the trade-off of bank regulation (see Bhattacharya et al. (1998) for the review of this literature). The contribution of our paper to explore the implications of the bond market for bank regulation from the general equilibrium and macro-prudential perspectives.

Thirdly, our paper contributes to a strand of macroeconomic literature that highlights the capital structure of firms such as De Fiore and Uhlig (2011), De Fiore and Uhlig (2015), Crouzet (2014). These papers model the surge in the cost of bank financing as an exogenous shock. Therefore, they could not have rich characterizations of dynamics of bank-financing and bond-financing as what we capture in our paper. In this regard, our paper is close to Rampini and Viswanathan (2015), which endogenize the cost of financial intermediation, although they does not either address the substitution between bank credit and bond credit or fully analyzes the dynamics of a stochastic economy.

Lastly, there are a large number of papers investigate the choice between bond-finance and bank-finance for firms in the corporate finance and banking literature (e.g., Chemmanur and Fulghieri (1994), Bolton and Freixas (2000)). The contribution of our paper to the literature is to highlight the dynamic properties of firms' capital structure and to explore the general equilibrium effects of firms' financing choices. In addition, our paper also stress that the cost of bank-financing fluctuates over business cycles and this has important effects on financial stability and economic growth.

The structure of the rest of the paper follows. Section 1 describes the set-up of the model and defines the equilibrium. In Section 2, we characterize the optimal choice of individual agents and the Markov equilibrium that this paper focuses on. Section 3 illustrates key properties of the Markov equilibrium with numerical examples. In Sections 4, we explore macroeconomic implication of the capital ratio requirement in our framework. Section 5 investigates the role of bond markets for optimal bank regulation. Lastly, Section 6 concludes the paper.

1 Model

In this section, we build a macro-finance model, in which firms can either directly issue corporate bonds or raise credit via financial intermediaries. The economy is infinite-horizon, continuous-time, and has two types of goods: perishable final goods and durable physical capital goods. Final goods serve as the numéraire.

Three groups of agents populate in the economy: experts, bankers, and households. All agents have the same logarithmic preferences and the time discount factor ρ. None of them accepts negative consumption. Although all three types of agents are able to hold physical capital goods and produce final goods, experts are most productive and bankers specialize in financial intermediation.

1.1 Technology

In each period, an expert can produce ak_t units of final goods with k_t units of physical capital. Households and bankers, who are less productive, also have linear production technologies: $y_t = a_h k_t$ for households and $y_t = a_b k_t$ for bankers, where $a_b < a_h < a$. All three types of agents can convert $\iota_t k_t$ units of final goods into $k_t \Phi(\iota_t)$ units of physical capital, where

$$\Phi(\iota_t) = \frac{\log(\iota_t \phi + 1)}{\phi}.$$

Thus, there is technological illiquidity on the production side. Physical capital in the possession of experts depreciates at rate δ and, in the hands of households and bankers, physical capital depreciates at rate δ_h and δ_b respectively.

Exogenous *aggregate* shocks are driven by a standard Brownian motion $\{Z_t, t \geq 0\}$. In the absence of any idiosyncratic shock, physical capital managed by an expert evolves according to

$$\mathrm{d}k_t = (\Phi(\iota_t) - \delta)k_t \mathrm{d}t + \sigma k_t \mathrm{d}Z_t. \tag{1}$$

Similarly, physical capital managed by households and bankers follows

$$\mathrm{d}k_t = (\Phi(\iota_t) - \delta_h)k_t \mathrm{d}t + \sigma k_t \mathrm{d}Z_t,$$

and

$$\mathrm{d}k_t = (\Phi(\iota_t) - \delta_b)k_t \mathrm{d}t + \sigma k_t \mathrm{d}Z_t,$$

In the beginning of each period, an expert becomes a *safe* one with probability α or a *risky* one with probability $1 - \alpha$. Whether an expert becomes risky within a period is independent across the time. Within a period, an adverse public signal may occur to a risky firm (a firm that a risky expert manages) with probability λ at an interim stage after the firm has made its investment, production, and financing decisions. The adverse signal implies that the quality of a firm's assets is under question and the firm owner can take advantage of its creditors because they have less inside information. Naturally, risky experts establish an infinitely number of firms to diversify this idiosyncratic risk. Safe firms do not experience such adverse signals.

1.2 Corporate Bond, Bank Loan, and Liquidation

A firm can raise credit either from issuing corporate bond or from a bank. In addition, we assume that no firm can issue outside equity and that all firms have limited liability.

Both corporate bonds and bank loans are collateralized contingent debt. Collateralized borrowing implies that if a firm raises L units of capital from creditors it must put down physical capital worth of L as collateral. If an adverse signal occurs to a risky firm at the interim stage, we assume that creditors of the firm always find it optimal to seize the collateral and liquidate physical capital.[1]

Bondholders are assumed to be less efficient than banks in terms of liquidating physical capital. This is because it is harder and more time-consuming to achieve a collective decision

[1] The micro-foundation for creditors' optimal decision is the following. Given the adverse signal, the quality of collateral is questionable and becomes unclear. As a result, it becomes easier for the firm owner to steal the collateral and nothing could be left to creditors. Therefore, the optimal decision for creditors is to seize the collateral given the negative signal.

for a number of bondholders during the liquidation process than it is for a single bank. In particular, we assume that the depreciation rate of physical capital rises to $\kappa^d + \delta$ if bondholders seize the collateral and the depreciation rate becomes $\kappa + \delta$ if banks liquidate the collateral, where $\kappa < \kappa^d$.

For simplicity, we assume that there is a passive mutual fund that serves the intermediary in the corporate bond market. The fund charges its borrowers the risk-free rate plus the expected loss due to costly liquidation and promises the risk-free rate r_t to its investors. Any loss or profit that the mutual fund has is shared by all households (including experts) in proportion to their net worth.[2] Thus, the unit cost of bond-financing is $r_t + \lambda\kappa^d$ for a risky firm.

Similar to the mutual fund, banks raise funds from households and promise the risk-free rate r_t. Unlike the passive mutual fund, banks will ask for a risk premium because their equity capital is exposed to the aggregate risk. Overall, the unit borrowing cost for bank-financing is $r_t^\lambda + \lambda\kappa$.

No liquidation is involved if a firm is self-financed.

1.3 An Expert's Problem

We conjecture that the equilibrium price of physical capital follows

$$\mathrm{d}q_t = \mu_t^q q_t \mathrm{d}t + \sigma_t^q q_t \mathrm{d}Z_t,$$

then the rate of return from holding physical capital for an expert in the absence of any shock is

$$R_t \mathrm{d}t \equiv \Big(\frac{a - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q\Big)\mathrm{d}t.$$

Since costly liquidation does not happen to a safe expert, she raises external funds only through bond-financing and thus her dynamic budget constraint is

$$\frac{\mathrm{d}w_t}{w_t} = R_t \mathrm{d}t + (\sigma + \sigma_t^q)\mathrm{d}Z_t + b_t^0(R_t \mathrm{d}t + (\sigma + \sigma_t^q)\mathrm{d}Z_t - r_t \mathrm{d}t) - \frac{c_t}{w_t}\mathrm{d}t, \tag{2}$$

where b_t^0 is the bond-to-equity ratio. Without loss of generality, we drop the loss or benefit that the expert takes from the mutual fund.

A risky expert will choose the financing method for his firms: corporate debt, bank

[2] In the interest of clear illustration, we do not include agents' exposure to the aggregate risk via the passive mutual fund in the following equations, although we take into account these effects when calculating the equilibrium.

loans, or self-financing. Since all of the expert's firms are identical prior to the realization of the liquidity shock, financing decisions of all firms managed by the expert are the same. Thus, the debt-to-equity ratio of these firms is also the same, which is exactly the expert's debt-to-net-worth ratio. The law of motion for the risky expert's net worth is

$$\begin{aligned}\frac{\mathrm{d}w_t}{w_t} &= R_t\mathrm{d}t + (\sigma + \sigma_t^q)\mathrm{d}Z_t + b_t^\lambda\Big(\big(R_t - \lambda\kappa^d - r_t\big)\mathrm{d}t + \big(1-\lambda\big)(\sigma+\sigma_t^q)\mathrm{d}Z_t\Big)\\ &\quad + l_t\Big(\big(R_t - \lambda\kappa - r_t^\lambda\big)\mathrm{d}t + \big(1-\lambda\big)(\sigma+\sigma_t^q)\mathrm{d}Z_t\Big) - \frac{c_t}{w_t}\mathrm{d}t \end{aligned}\tag{3}$$

where b_t^λ is firms' bond-to-equity ratio and l_t firm's loan-to-equity ratio. By the Law of Large Numbers, the adverse signal at the interim stage implies that creditors seizes λ proportion of the expert's physical capital. As a result, the risky expert partially unloads his exposure to the aggregate risk, $\lambda(\sigma + \sigma_t^q)\mathrm{d}Z_t$.

Taking $\{q_t, r_t, r_t^\lambda, t \geq 0\}$ as given, an expert chooses $\{c_t, b_t^0, b_t^\lambda, l_t, t \geq 0\}$ to maximize her life-time expected utility

$$E_0\left[\int_0^\infty e^{-\rho t}\ln(c_t)\,\mathrm{d}t\right],\tag{4}$$

given that his net worth evolves in each period according to either equation (2) or (3) depending on her type in a period.

1.4 A Banker's Problem

The instant rate of return from holding physical capital for a banker is

$$R_t^b\mathrm{d}t \equiv \Big(\frac{a_b - \iota_t}{q_t} + \Phi(\iota_t) - \delta_b + \mu_t^q + \sigma\sigma_t^q\Big)\mathrm{d}t.$$

Therefore, a banker's net worth n_t evolves according to

$$\frac{\mathrm{d}n_t}{n_t} = x_t^j\big(R_t^b\mathrm{d}t + (\sigma+\sigma_t^q)\mathrm{d}Z_t\big) + x_t\big(r_t^\lambda\mathrm{d}t + \lambda(\sigma+\sigma_t^q)\mathrm{d}Z_t\big) + (1 - x_t^j - x_t)r_t\mathrm{d}t - \frac{c_t}{n_t}\mathrm{d}t,\tag{5}$$

where x_t^j denotes the physical-capital-to-equity ratio and x_t the loan-to-equity ratio for the bank. When $x_t > 1$, the bank absorbs deposits and intermediates funds from households to experts. When $x_t \leq 1$, the bank saves some of its equity capital in the mutual fund. The banker is exposed to the aggregate risk $x_t^\lambda\lambda(\sigma + \sigma_t^q)\mathrm{d}Z_t$ because she takes over and resell the physical capital that backs her lending. Taking $\{q_t, r_t, r_t^\lambda, t \geq 0\}$ as given, a banker chooses

$\{c_t, x_t^j, x_t^\lambda, t \geq 0\}$ to maximize her life-time expected utility

$$E_0\left[\int_0^\infty e^{-\rho t}\ln(c_t)\right] \tag{6}$$

subject to the dynamic budget constraint (5).

1.5 A Household's Problem

The rate of return from holding physical capital for a household in the absence of any shock is

$$R_t^h \mathrm{d}t \equiv \left(\frac{a_h - \iota_t}{q_t} + \Phi(\iota_t) - \delta_h + \mu_t^q + \sigma\sigma_t^q\right)\mathrm{d}t,$$

which is similar to the corresponding term for experts R_t. The law of motion for a household's net worth w_t^h is

$$\frac{\mathrm{d}w_t^h}{w_t^h} = x_t^h(R_t^h \mathrm{d}t + (\sigma + \sigma_t^q)\mathrm{d}Z_t) + (1 - x_t^h)r_t \mathrm{d}t - \frac{c_t}{w_t^h}\mathrm{d}t, \tag{7}$$

where x_t^h is the portfolio weight of physical capital. Taking $\{q_t, r_t, t \geq 0\}$ as given, a households maximize his life-time expected utility

$$E_0\left[\int_0^\infty e^{-\rho t}\ln(c_t)\right] \tag{8}$$

by choosing $\{c_t, x_t^h, t \geq 0\}$ that satisfy the dynamic budget constraint (7).

1.6 Equilibrium

The aggregate shock $\{Z_t, t \geq 0\}$ drives the evolution of the economy. $\mathbf{I} = [0, 1)$ denotes the set of experts, $\mathbf{J} = [1, 2)$ the set of bankers, and $\mathbf{H} = [2, 3]$ the set of households. Given the idiosyncratic shock in period t, $\mathbf{I}_t^s$ is the set of safe experts in period t and $\mathbf{I}_t^r$ the set of risky experts.

Definition 1 *Given the initial endowments of physical capital* $\left\{k_0^i, k_0^j, k_0^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\right\}$ *to experts, bankers, and households such that*

$$\int_0^1 k_0^i \mathrm{d}i + \int_1^2 k_0^j \mathrm{d}j + \int_2^3 k_0^h \mathrm{d}h = K_0,$$

an equilibrium is defined by a set of stochastic processes adapted to the filtration generated by $\{Z_t\}_{t=0}^\infty$*: the price of physical capital* $\{q_t\}_{t=0}^\infty$*, risk-free rate* $\{r_t\}_{t=0}^\infty$*, the interest* *rate of bank loan* $\{r_t^\lambda\}_{t=0}^\infty$, *wealth* $\{W_t^i, N_t^j, W_t^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\}_{t=0}^\infty$, *investment decisions* $\{\iota_t^i, \iota_t^j, \iota_t^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\}_{t=0}^\infty$, *asset holding decisions* $\{x_t^j, x_t^h, j \in \mathbf{J}, h \in \mathbf{I}_t^h\}_{t=0}^\infty$ *of bankers and households, corporate debt financing decisions* $\left\{b_t^{i,0}, b_t^{i,\lambda}, i \in \mathbf{I}_t\right\}_{t=0}^\infty$ *of experts, bank financing decisions* $\{l_t^i, i \in \mathbf{I}_t^r\}_{t=0}^\infty$ *of risky experts, bank lending* $\{x_t^{\lambda,j}, j \in \mathbf{J}\}_{t=0}^\infty$ *and consumption* $\{c_t^i, c_t^j, c_t^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\}_{t=0}^\infty$*; such that*

1. $W_0^i = k_0^i q_0$, $N_0^j = k_0^j q_0$, *and* $W_0^h = k_0^h q_0$ *for* $i \in \mathbf{I}$, $j \in \mathbf{J}$, *and* $h \in \mathbf{H}$*;*
2. *each expert, banker, and household solve for their problems given prices;*
3. *markets for final goods and physical capital clear, that is,*

$$\begin{aligned}\int_0^3 c_t^i \mathrm{d}i &= \frac{1}{q_t}\int_1^2 (a^b - \iota_t^j) n_t^j x_t^j \mathrm{d}j + \frac{1}{q_t}\int_2^3 (a^h - \iota_t^h) w_t^h x_t^h \mathrm{d}h + \\ &\quad \frac{1}{q_t}\int_{i\in\mathbf{I}_t^s} \left(a - \iota_t^i\right) w_t^i (1 + b_t^{i,0}) \mathrm{d}i + \frac{1}{q_t}\int_{i\in\mathbf{I}_t^r} \left(a - \iota_t^i\right) w_t^i (1 + b_t^{i,\lambda} + l_t^i) \mathrm{d}i\end{aligned}$$

for the market of final goods, and

$$\frac{1}{q_t}\int_{i\in\mathbf{I}_t^s} w_t^i (1 + b_t^{i,0}) \mathrm{d}i + \frac{1}{q_t}\int_{i\in\mathbf{I}_t^r} w_t^i (1 + b_t^{i,\lambda} + l_t^i) \mathrm{d}i + \frac{1}{q_t}\int_1^2 n_t^j x_t^j \mathrm{d}j + \frac{1}{q_t}\int_2^3 w_t^h x_t^h \mathrm{d}h = K_t$$

for the market of physical capital goods, where K_t *evolves according to*

$$\begin{aligned}\frac{\mathrm{d}K_t}{\mathrm{d}t} &= \frac{1}{q_t}\int_1^2 \left(\Phi(\iota_t^j) - \delta^b\right) n_t^j x_t^j \mathrm{d}j + \frac{1}{q_t}\int_2^3 \left(\Phi(\iota_t^h) - \delta^h\right) w_t^h x_t^h \mathrm{d}h \\ &+ \frac{1}{q_t}\int_{i\in\mathbf{I}_t^s} \left(\Phi(\iota_t^i) - \delta\right) w_t^i (1 + b_t^{i,0})\, \mathrm{d}i \\ &+ \frac{1}{q_t}\int_{i\in\mathbf{I}_t^r} \left(\Phi(\iota_t^i) + \delta^r - \delta\right) w_t^i (1 + b_t^{i,\lambda} + l_t^i) - \lambda\kappa^d w_t^i b_t^i - \lambda\kappa w_t^i l_t^i\, \mathrm{d}i.\end{aligned}$$

4. *the bank loan market clears*

$$\int_{i\in\mathbf{I}_t^r} w_t^i l_t^i \mathrm{d}i = \int_1^2 n_t^j x_t^{\lambda,j} \mathrm{d}j.$$

The credit market for corporate debt clears automatically by Walras' Law.

2 Solving for the Equilibrium

Both experts' net worth and bank capital are crucial for the allocation of physical capital and financial resources in the equilibrium. We expect that the price of physical capital declines as the share of both experts' net worth bank capital shrinks due to adverse exogenous shocks.

To solve for the equilibrium, we first derive first-order conditions with respect to optimal decisions of experts, bankers, and households; secondly, we solve for the law of motion for endogenous state variables, wealth shares of different groups of agents, based on market clearing conditions as well as first-order conditions; lastly, we use first-order conditions and state variables' law of motion to define partial differential equations that endogenous variables such as the price of physical capital satisfy.

2.1 Households' Optimal Choices

Households have logarithmic preferences. In the following discussion, we will take advantage of two well-known properties with respect to logarithmic preferences in the continuous-time setting: 1) a household's consumption c_t is ρ proportion of her wealth w_t^h in the same period, i.e.,

$$c_t = \rho w_t^h; \tag{9}$$

2) a household's portfolio weight on a risky investment is such that the Sharpe ratio of the risky investment equals the percentage volatility of her wealth.

A household's investment rate ι_t always maximizes $\Phi(\iota_t) - \iota_t/q_t$. The first-order condition implies that

$$\Phi'(\iota_t) = \frac{1}{q_t}, \tag{10}$$

which defines the optimal investment as a function of the price of physical capital $\iota(q_i)$.

Given the second property discussed above, it is straightforward to derive a household's optimal portfolio weight on the physical capital x_t^h, which satisfies [3]

$$x_t^h = \frac{\max\{R_t^h - r_t, 0\}}{(\sigma + \sigma_t^q)^2}. \tag{11}$$

2.2 Experts' Portfolio Choices

According to the second property highlighted above, it is straightforward to characterize a safe expert's optimal bond-to-equity ratio[4]

$$b_t^0 = \frac{\max\{R_t - r_t - (\sigma + \sigma_t^q)^2, 0\}}{(\sigma + \sigma_t^q)^2}. \tag{12}$$

[3] Given that $R_t^h > r_t$, Sharpe ratio is ${}^{(R_t^h - r_t)}/_{(\sigma+\sigma_t^q)}$. The percentage volatility of the household's wealth is $x_t^h(\sigma + \sigma_t^q)$. Hence the optimal x_t^h is such that $x_t^h(\sigma + \sigma_t^q) = {}^{(R_t^h - r_t)}/_{(\sigma+\sigma_t^q)}$.

[4] In this case, Sharpe ratio is ${}^{(R_t - r_t)}/_{(\sigma+\sigma_t^q)}$. The percentage volatility of the safe expert's wealth is $(1 + b_t^0)(\sigma + \sigma_t^q)$.

For a risky expert, both bond-to-equity ratio b_t^λ and loan-to-equity ratio l_t affect the percentage volatility of her wealth $(1+(1-\lambda)b_t^\lambda+(1-\lambda)l_t)(\sigma+\sigma_t^q)$. Hence, optimal b_t^λ and l_t must satisfy

$$\frac{R-\lambda\kappa^d-r_t}{(1-\lambda)(\sigma+\sigma_t^q)} = (1+(1-\lambda)b_t^\lambda+(1-\lambda)l_t)(\sigma+\sigma_t^q)$$
$$\frac{R-\lambda\kappa-r_t^\lambda}{(1-\lambda)(\sigma+\sigma_t^q)} = (1+(1-\lambda)b_t^\lambda+(1-\lambda)l_t)(\sigma+\sigma_t^q),$$

if $b_t^\lambda > 0$ and $l_t > 0$. Nevertheless, if the spread between the loan rate and the risk-free rate $r_t^\lambda - r_t$ is sufficiently small, it is possible that bank-financing strictly dominates bond-financing since $\kappa^d > \kappa$ and thus $b_t^\lambda = 0$. Therefore, first-order conditions for optimal b_t^λ and l_t are

$$\frac{R-\lambda\kappa^d-r_t}{(1-\lambda)(\sigma+\sigma_t^q)} \leq (1+(1-\lambda)b_t^\lambda+(1-\lambda)l_t)(\sigma+\sigma_t^q), \text{with equality if } b_t^\lambda > 0; \tag{13}$$
$$\frac{R-\lambda\kappa-r_t^\lambda}{(1-\lambda)(\sigma+\sigma_t^q)} \leq (1+(1-\lambda)b_t^\lambda+(1-\lambda)l_t)(\sigma+\sigma_t^q), \text{with equality if } l_t > 0. \tag{14}$$

When the cost of bond-financing equals the cost of bank-financing, i.e., $\lambda\kappa^d+r_t = \lambda\kappa+r_t^\lambda$, individual risky experts are indifferent between bond-financing and bank-financing and their portfolio choices are indeterminate. Without loss of generality, we assume that portfolio weights of both bond-financing and bank-financing, b_t^λ and l_t, are the same across all risky experts.

2.3 Banker's Optimal Choices

A banker's optimal portfolio weights on holding of physical capital and loans satisfy

$$R_t^b - r_t = (x_t^j+\lambda x_t)(\sigma+\sigma_t^q)^2,$$

and

$$r_t^\lambda - r_t = \lambda(x_t^j+\lambda x_t)(\sigma+\sigma_t^q)^2. \tag{15}$$

Loan rate r_t^λ relies on banks' exposure to aggregate risk $\lambda(\sigma+\sigma_t^q)$ and banks' leverage x_t and x_t^j. The financing cost of bank loans for firms fluctuates endogenously not just because the price volatility of physical capital changes over time but also because banks' leverage varies across business cycles.

2.4 Market Clearing

Let W_t denote the total wealth that experts have in period t and N_t the total bank capital. Hence, the total bank loans issued in equilibrium denoted by $x_t N_t$ satisfies

$$x_t N_t = (1-\alpha) W_t l_t. \tag{16}$$

The demand for final goods consists of consumption, intermediation costs, and investments. The aggregate consumption of households is $\rho q_t K_t$. The total intermediation cost is $\tau x_t N_t \mathbf{1}_{x_t > 1}$. Therefore, the market clearing condition with respect to final goods is

$$\begin{aligned}\rho q_t K_t = \alpha \frac{W_t}{q_t}(a - \iota_t)(1 + b_t^0) + (1-\alpha)\frac{W_t}{q_t}(a - \iota_t)(1 + b_t^\lambda + l_t) \\ + \frac{N_t}{q_t}(a_b - \iota_t)x_t^j + \frac{q_t K_t - W_t - N_t}{q_t}(a_h - \iota_t)x_t^h\end{aligned} \tag{17}$$

Finally, the market for physical capital clears if

$$\alpha \frac{W_t}{q_t}(1 + b_t^0) + (1-\alpha)\frac{W_t}{q_t}(1 + b_t^\lambda + l_t) + \frac{N_t}{q_t}x_t^j + \frac{q_t K_t - W_t - N_t}{q_t}x_t^h = K_t. \tag{18}$$

2.5 Wealth Distribution

Wealth shares of both experts and bankers matter for the equilibrium. Two endogenous state variables that characterize the dynamics of the economy are experts' wealth share $\omega_t = {W_t}/{(q_t K_t)}$ and bankers' wealth share $\eta_t = {N_t}/{(q_t K_t)}$.

The decline of experts' wealth share naturally leads to the fall of average productivity since financial markets are imperfect and households are less productive. If bankers' wealth share declines, then the supply of bank loans shrinks and the interest rate on bank loans rises, which, in turn, also lowers the aggregate productivity of the economy the increased cost of raising external finance for experts.

Given dynamic budget constraints of individual experts and bankers, it is straightforward to derive laws of motion for both W_t and N_t

$$\begin{aligned}\frac{\mathrm{d}W_t}{W_t} = \Big(R_t + \alpha b_t^0(R_t - r_t) + (1-\alpha)b_t^\lambda(R_t + \delta^r - \lambda\kappa^d - r_t) + (1-\alpha)l_t(R_t + \delta^r - \lambda\kappa - r_t^\lambda)\Big)\mathrm{d}t \\ - \frac{c_t}{W_t}\mathrm{d}t + \Big(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)(1-\lambda)\Big)(\sigma + \sigma_t^q)\mathrm{d}Z_t\end{aligned} \tag{19}$$

$$\frac{\mathrm{d}N_t}{N_t} = \Big(x_t^j R_t^b + x_t r_t^\lambda + (1 - x_t^j - x_t)r_t - \frac{c_t}{N_t}\Big)\mathrm{d}t + (x_t^j + x_t\lambda)(\sigma + \sigma_t^q)\mathrm{d}Z_t \tag{20}$$

Dynamics of state variables in equilibrium also depend on the law of motion of the aggregate physical capital, which is

$$\begin{aligned} \frac{\mathrm{d}K_t}{K_t} &= \mu_t^K \mathrm{d}t + \sigma \mathrm{d}Z_t, \text{where} \\ \mu_t^K &\equiv \Phi(\iota_t) - \delta - (1-\omega_t-\eta_t)x_t(\delta-\delta^h) + (1-\alpha)\omega_t\big(b_t^\lambda \delta^r + l_t\delta^r - \lambda(b_t^\lambda \kappa^d + l_t\kappa)\big). \end{aligned} \tag{21}$$

Given laws of motion of W_t, N_t, q_t, and K_t, we can apply Ito's Lemma to derive laws of motion for ω_t and η_t in equilibrium, which are summarized in the following lemma.

Lemma 1 *In equilibrium, experts' wealth share ω_t evolves according to*

$$\frac{\mathrm{d}\omega_t}{\omega_t} = \mu_t^\omega \mathrm{d}t + \sigma_t^\omega \mathrm{d}Z_t, \tag{22}$$

, where

$$\begin{aligned} \mu_t^\omega &= R_t - \mu_t^q - \mu_t^K - \sigma\sigma_t^q + \alpha b_t^0(R_t - r_t) + (1-\alpha)b_t^\lambda(R_t\delta^r - \lambda\kappa^d - r_t^\lambda) \\ &\quad + (1-\alpha)l_t(R_t + \delta^r - \lambda\kappa - r_t^\lambda) - \big(\alpha b_t^0 + (1-\alpha)b_t^\lambda(1-\lambda) + (1-\alpha)l_t(1-\lambda)\big)(\sigma+\sigma_t^q)^2 - \rho \\ \sigma_t^\omega &= \big(\alpha b_t^0 + (1-\alpha)b_t^\lambda(1-\lambda) + (1-\alpha)l_t(1-\lambda)\big)(\sigma+\sigma_t^q). \end{aligned}$$

And, the state variable η_t evolves according to

$$\frac{\mathrm{d}\eta_t}{\eta} = \mu_t^\eta \mathrm{d}t + \sigma_t^\eta \mathrm{d}Z_t, \tag{23}$$

where

$$\begin{aligned} \mu_t^\eta &= (x_t^j + \lambda x_t)(x_t^j + \lambda x_t - 1)(\sigma+\sigma_t^q)^2 + r_t - \mu_t^q - \mu_t^K - \sigma\sigma_t^q + (\sigma+\sigma^q)^2 - \rho \\ \sigma_t^\eta &= (x_t^j + \lambda x_t - 1)(\sigma + \sigma_t^q) \end{aligned}$$

The proof of Lemma 1 is in appendix.

2.6 Markov Equilibrium

Like other continuous-time macro-finance models (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2012), our framework also has the property of scale-invariance with respect to total physical capital K_t. Thus, we will focus on the equilibrium that is Markov in state variables ω_t and η_t. In the Markov equilibrium, dynamics of endogenous variables such as q_t can be characterized by laws of motion of ω_t and η_t and functions $q(\omega, \eta)$.

To solve for full dynamics of the economy, we derive a partial differential equations with respect to $q(\omega, \eta)$. The partial differential equation as well as its boundary conditions originates from equilibrium conditions and Ito's formula with $q(\omega, \eta)$. Ito's lemma with respect to the volatility of the price of physical capital implies that

$$q_t \sigma_t^q = q_\omega(\omega_t, \eta_t)\omega_t \sigma_t^\omega + q_\eta(\omega_t, \eta_t)\eta_t \sigma_t^\eta. \tag{24}$$

Given (q, ω, η), we can solve the equilibrium and derive all endogenous choice variables $(c, b^0, b^\lambda, l, x, x^h)$ and endogenous price variables $(r, r^\lambda, \mu^q, \sigma^q)$.[5] Therefore, volatility terms of two state variables $(\sigma^\eta, \sigma^\omega)$ are also known. Hence, equation (24) is a well-defined partial differential equation with respect to $q(\omega, \eta)$.

In addition to the differential equation, we need boundary conditions to solve for $q(\omega, \eta)$. There are three boundary conditions that correspond to three boundaries for the domain of $q(\omega, \eta)$: $\{(\omega, \eta) : \omega = 0, 0 \leq \eta \leq 1\}$, $\{(\omega, \eta) : 0 \leq \omega \leq 1, \eta = 0\}$, and $\{(\omega, \eta) : 0 \leq \omega \leq 1, 0 \leq \eta \leq 1, \omega + \eta = 1\}$. For any of the three boundaries, one of the three agents has zero net worth and the economy becomes one with only two types of agents. Accordingly, differential equation (24) on boundaries reduces to an ordinary differential equation, which is straightforward to characterize.

3 Results

In this section, we discuss main results of the model with numerical examples. The choice of parameter values is $\rho = 3\%$, $a = 0.16$, $a^h = 0$, $a^b = 0$, $\delta = 0.01$, $\delta^b = 0.1$, $\delta^h = 0.01$, $\phi = 5$, $\alpha = 0.2$, $\lambda = 0.3$, $\kappa^d = 0.4$, $\kappa = 0.2$, and $\sigma = 0.1$.

The dynamics of the economy are fully governed by the law of motion of the two state variables (ω, η), equations (22) and (23). In this case, we will characterize the equilibrium by presenting endogenous variables such as the price of physical capital as functions of the two state variables.

3.1 Price and the Misallocation of Physical Capital

The misallocation of physical capital exists because productive experts cannot issue outside equity and the use of leverage expose them to the risk that their net worth could be completely wiped out. Therefore, when experts' wealth share is arbitrarily close to zero experts

[5] At this stage given only (q, ω, η), we can only solve for $r - \mu^q$. However, it does not cause any problem for solving for $q(\omega, \eta)$.

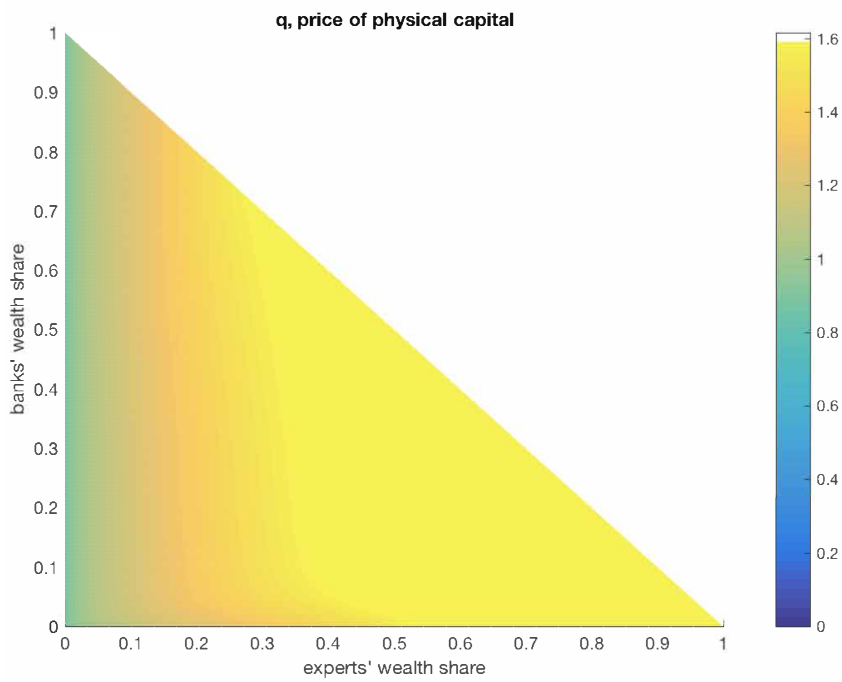

Figure 1: The price of physical capital. This figure shows the price of physical capital $q(\omega, \eta)$ as a function of two state variables: experts' wealth share and bankers' wealth share. The colar bar on the right indicates the value of the function in a given state. For parameter values, see the beginning of Section 3.

only hold a small fraction of physical capital in the economy (Figure 2) and the price of physical capital converges to its lower bound $q_{min} = 0.8696$ (Figure 1). Given the same level of experts' wealth share, the price of physical capital declines as bankers' wealth share drops (Figure 1). This is because the supply of bank loans becomes smaller if the banking sector is less capitalized. Hence, risky experts find it harder to raise external capital and the misallocation of physical capital becomes more severe (Figure 2).

3.2 Endogenous Risk and Amplification Mechanism

The exogenous Brownian shocks hit both experts' net worth and bank capital in the economy. The impact of the exogenous shock is amplified through the following two inter-connected vicious spirals. The decline in experts' net worth lowers their holdings of physical capital, which depresses its price and hurts experts' net worth. In addition, the decline in bank capital raises the cost of obtaining bank loans and deter risky firms from raising external funds. And, this also lowers the aggregate productivity and pushes down the price of physical capital, which in turn impairs net worth of both expert and banker sectors further.

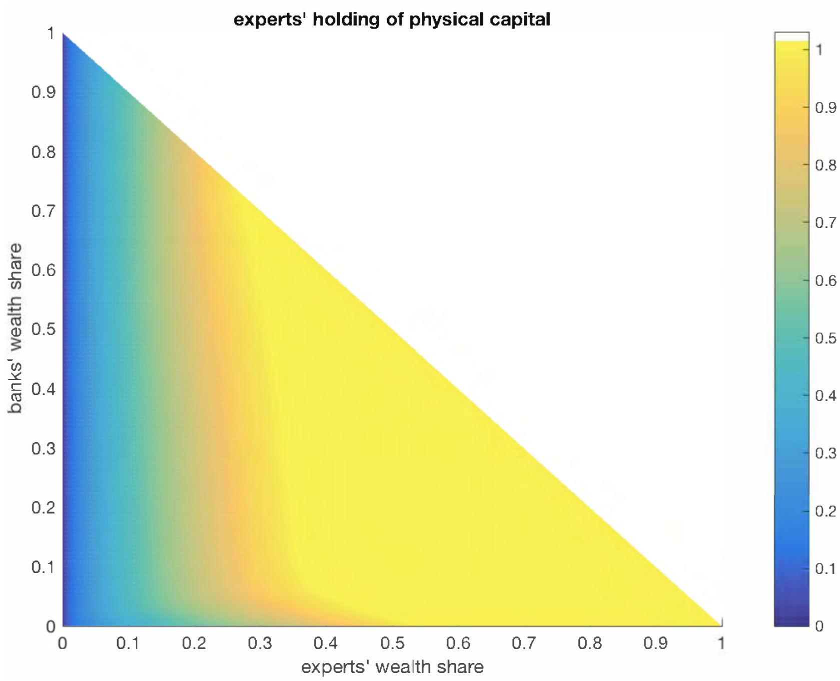

Figure 2: The fraction of physical capital that experts hold. This figure shows the fraction of physical capital that experts hold $\psi(\omega, \eta)$ as a function of two state variables: experts' wealth share and bankers' wealth share. The colar bar on the right indicates the value of the function in a given state. For parameter values, see the beginning of Section 3.

To clearly illustration the amplification mechanism, we rewrite equation (24)

$$q\sigma^q = \frac{q_\omega \omega(\alpha b^0 + (1-\alpha)b^\lambda(1-\lambda) + (1-\alpha)l(1-\lambda)) + q_\eta \eta(x^j + \lambda x - 1)}{1 - \frac{q_\omega}{q}\omega(\alpha b^0 + (1-\alpha)b^\lambda(1-\lambda) - (1-\alpha)l(1-\lambda)) - \frac{q_\eta}{q}\eta(x^j + \lambda x - 1)}\sigma. \tag{25}$$

We can see that the magnitude of endogenous risk depends on i) the sensitivity of the price of physical capital to the change of wealth shares of experts and bankers, q_ω and q_η; ii) the exposure of their wealth shares to the aggregate risk.

Figure 3 indicates that endogenous risk is low when experts hold all physical capital in the economy (Figure 2). When asset fire sales occur, endogenous risk increases. Figure 3 also shows that the economy is extremely unstable when the total wealth of the economy is concentrated in the banking sector and the productive sector only possesses a slim fraction of total wealth in the economy (the upper left region of Figure 3). The underlying reason of this result is straightforward. Endogenous risk originates from the risk of asset fire-sales, which ultimately depends on the net worth of the productive sector (Figure 5) instead of the intermediary sector (Figure 4). Our result highlights that an overly-capitalized financial intermediary sector could be harmful for financial stability and also problematic for the entire economy.

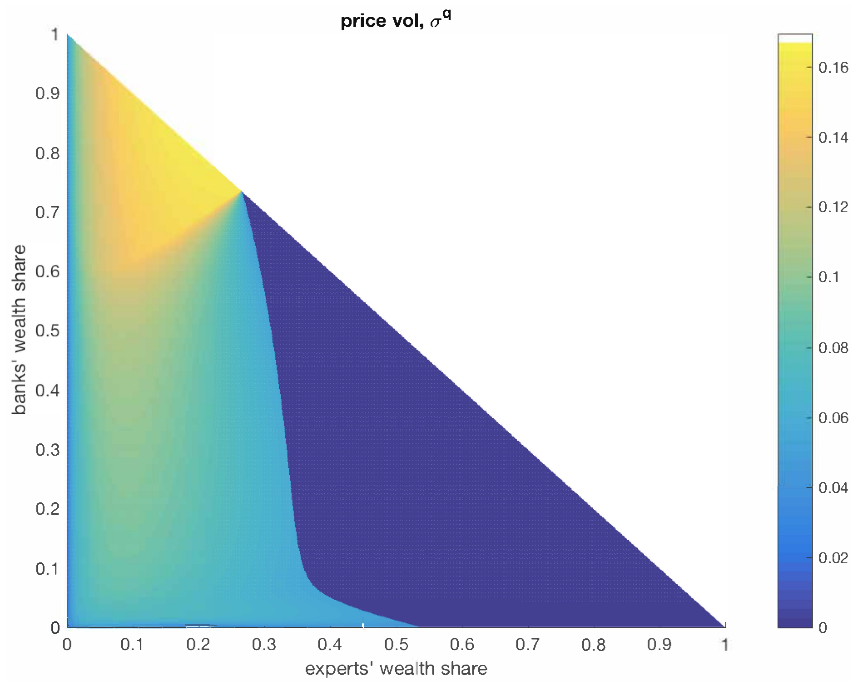

Figure 3: The volatility of the price of physical capital. This figure shows the volatility of the price of physical capital $q(\omega, \eta)\sigma^q(\omega, \eta)$ as a function of two state variables: experts' wealth share and bankers' wealth share. The colar bar on the right indicates the value of the function in a given state. For parameter values, see the beginning of Section 3.

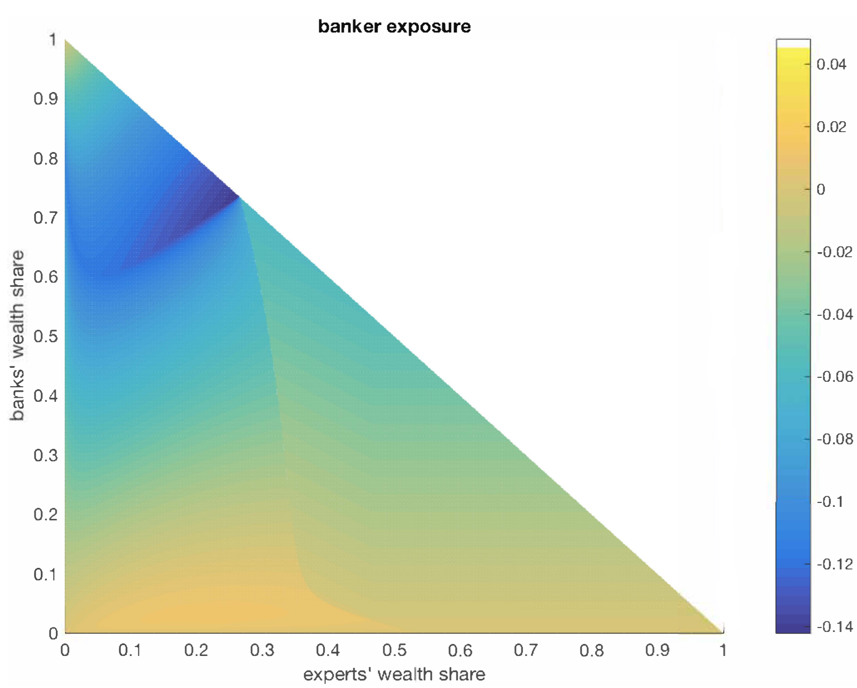

Figure 4: Bankers' exposure to the aggregate risk. This figure shows bankers' exposure to the aggregate risk $\omega\sigma^\omega(\omega, \eta)$ as a function of two state variables: experts' wealth share and bankers' wealth share. The colar bar on the right indicates the value of the function in a given state. For parameter values, see the beginning of Section 3.

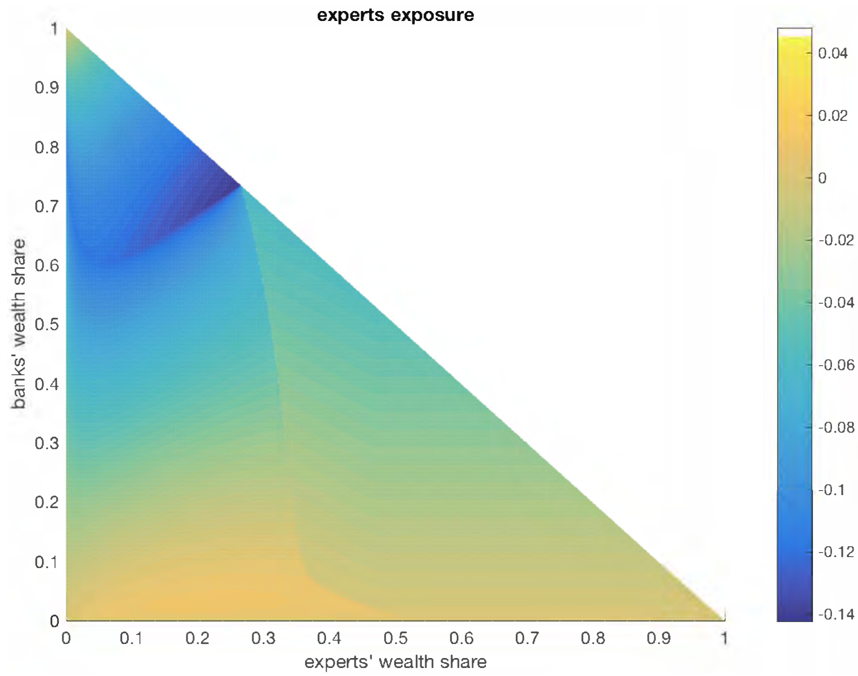

Figure 5: Experts' exposure to the aggregate risk. This figure shows experts' exposure to the aggregate risk $\eta\sigma^{\eta}(\omega, \eta)$ as a function of two state variables: experts' wealth share and bankers' wealth share. The colar bar on the right indicates the value of the function in a given state. For parameter values, see the beginning of Section 3.

3.3 Endogenous Fluctuation of Intermediation Costs

Costs of both bond-financing and bank-financing consist of two components: the cost of liquidation and the interest rate charged by creditors. Bank-financing dominates bond-financing in terms of the cost of liquidation, $\lambda\kappa < \lambda\kappa^d$. With respect to the interest payment, firms only pay the risk-free rate for issuing corporate debt regardless of their risks. In contrast, raising external funds from banks involves compensating banks for their exposures to both exogenous risk and endogenous risk, $\lambda(x_t^j + \lambda x_t)\big(\sigma + \sigma_t^q\big)^2$. Recall

$$r_t^{\lambda} - r_t = \lambda(x_t^j + \lambda x_t)\big(\sigma + \sigma_t^q\big)^2.$$

One particular feature of bank-financing in our model is that its cost fluctuates endogenously in the dynamics of the economy. Bankers who are financial intermediaries in the economy channel funds provided byhouseholds to more productive experts. However, financial intermediaries cannot issue outside equity to households due to asymmetric information problem modelled in papers such as He and Krishnamurthy (2012) and Brunnermeier and Sannikov (2014). As a result, bankers can only issue risk-free debt to households. The interest rate spread $r_t^{\lambda} - r_t$ that financial intermediaries earn from loans made to risky firms

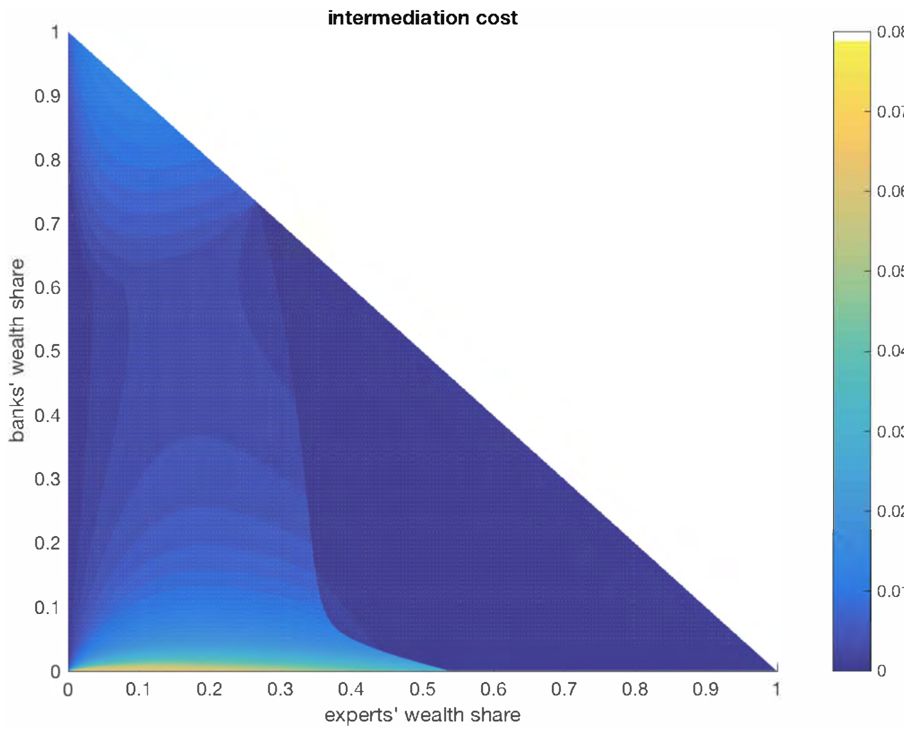

Figure 6: Intermediation cost $r^{\lambda} - r$. This figure shows the intermediation cost $r^{\lambda} - r$ as a function of two state variables: experts' wealth share and bankers' wealth share. The colar bar on the right indicates the value of the function in a given state. For parameter values, see the beginning of Section 3.

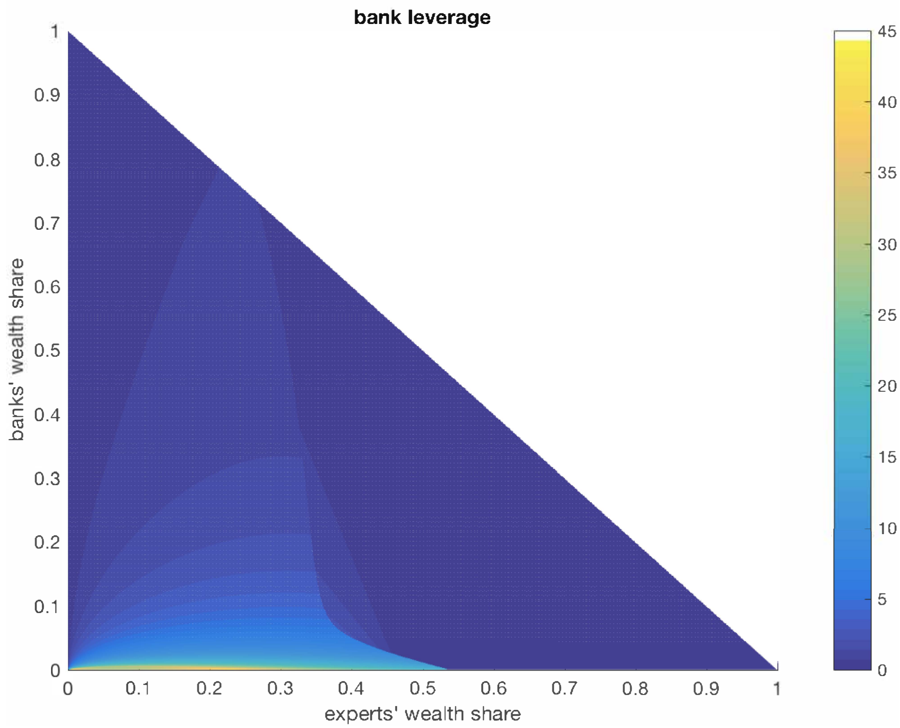

Figure 7: Bank leverage x. This figure shows banks' leverage $x(\omega, \eta)$ as a function of two state variables: experts' wealth share and bankers' wealth share. The colar bar on the right indicates the value of the function in a given state. For parameter values, see the beginning of Section 3.

depends on three components: banks' leverage, x_t, the exposure to a risky firm's credit event λ, and the magnitude of endogenous risk σ_t^q. When the banking sector is well capitalized, it is relatively resilient to adverse exogenous shocks. Hence, both x_t and σ_t^q are small in economic booms, and thus risky firms find it more profitable to raise credit from banks in economic upturns. In downturns, however, when the banking sector is not financially healthy, banks become less tolerant of taking risks and endogenous risk also goes up. Overall, the rise in the cost of bank-financing in downturns squeezes risky firms to more costly bond-financing or self-financing, which of course hurts the aggregate productivity.

Figure 6 indicates that *i*) the intermediation cost is high when the banking sector is poorly capitalized (lower left areas of both Figure 6 and 7) and *ii*) when high endogenous risk also leads to elevated intermediation cost even when the banking sector is overly-capitalized (upper left area of Figure 6). In addition, Figure 7 shows that bank leverage is counter-cyclical, which is standard in the literature.

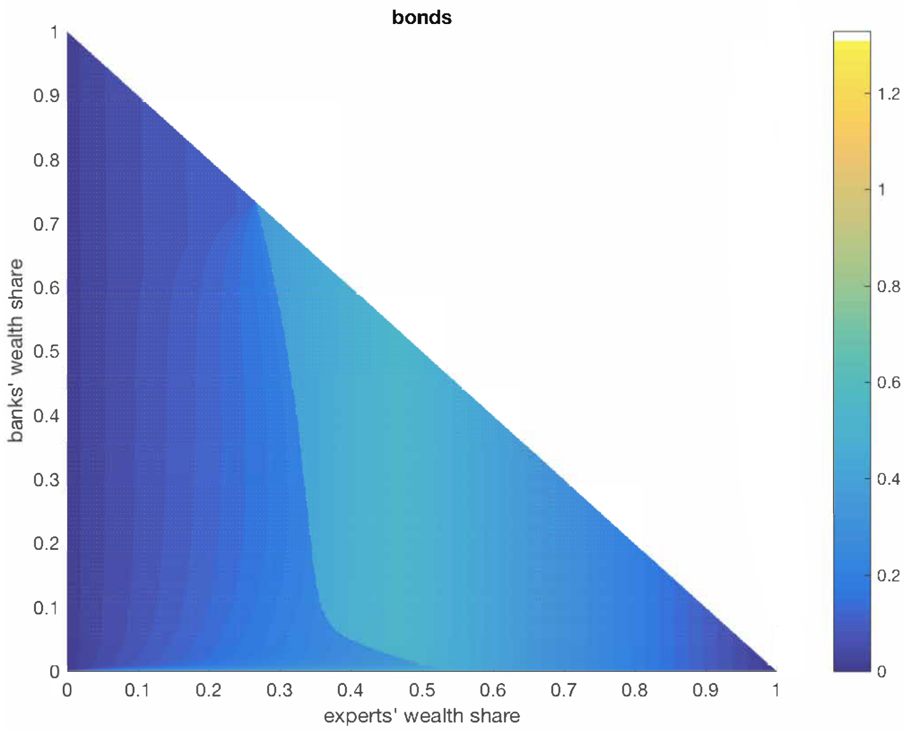

Figure 8: Outstanding corporate bonds as a fraction of total wealth in the economy. The colar bar on the right indicates the value of the function in a given state. For parameter values, see the beginning of Section 3.

3.4 Heterogeneity of Bond-Financing and Bank-Financing

Bond-financing is acyclical in our model Figure 8. As the economy evolves into economic booms, the share of outstanding corporate debt in total wealth goes up. This is primarily the consequence of safe firms' high debt-to-equity ratio due to low endogenous risks. Our paper highlights that the credit market of direct finance can also benefit from the development of the financial intermediary sector. In economic downturns, the share of corporate debt is also high because 1) risky firms switch to bond-financing due to the rising cost of bank-financing, and 2) firms take high leverage due to high returns from holding physical capital.

In contrast, Panel b in Figure 9 shows that bank-financing is clearly pro-cyclical. This is true since bank-financing is pro-cyclical at both intensive margin and extensive margin: all risky firms choose bank-financing and they take high leverage in economic booms when endogenous risk is low.

The substitution of bond credit for bank credit in economic downturns has significant price effects in equilibrium. When bank loans are very expensive, risky firms have to replace bank credit with bond-financing. Noticing that bond-financing involves more costly liquidation than bank-financing does, the rising borrowing cost for firms exerts downward pressure

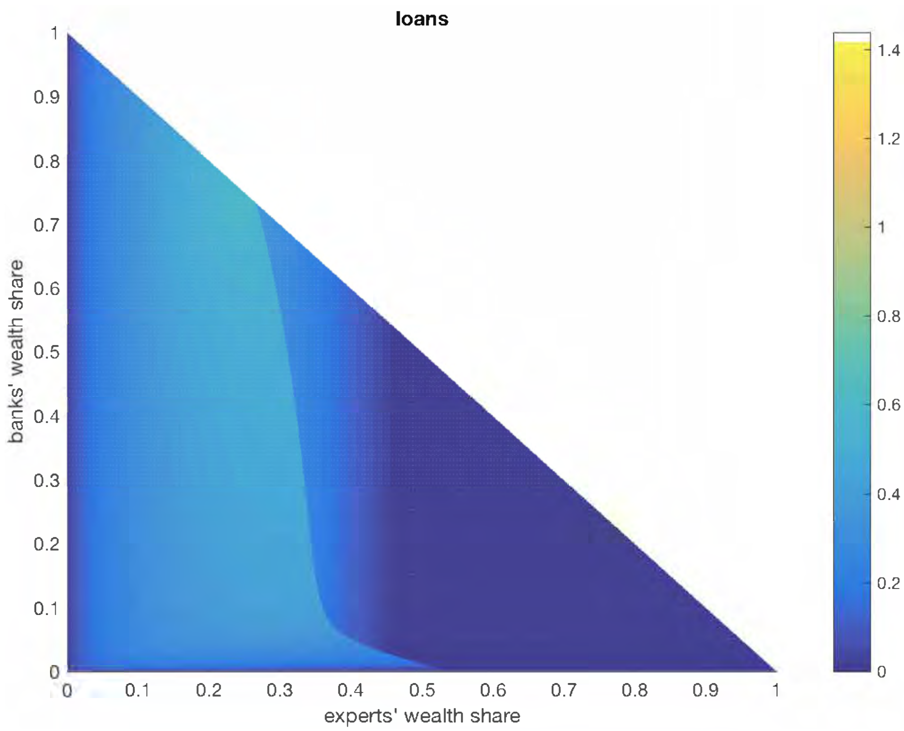

Figure 9: Outstanding bank loans as a fraction of total wealth in the economy. The colar bar on the right indicates the value of the function in a given state. For parameter values, see the beginning of Section 3.

on the price of physical capital. This explains why the magnitude of endogenous risks goes up when a large proportion of firms replacing bank credit with bond credit.

Overall, our model accounts for two facts of bond-financing and bank-financing in business cycles. The first fact is that bank-financing is more volatile and cyclical than bond-financing in the long-run as Becker and Ivashina (2014) document. The second fact, which Adrian et al. (2012) and many other papers have highlighted, is that the drastic decline in intermediated finance during big recessions such as 2007-09 financial crisis is partially made up by the increase in direct finance.

The reason why our model can capture the two facts has to do with two features of our framework: a feature on the technical side and a feature on the economics side. The technical feature is that our continuous-time frame allows for the full characterization of the dynamics of the economy. Thus, we do not only know the property of the equilibrium around the steady state but also we can precisely observe the equilibrium outcome in extreme states. Sometimes, properties of the equilibrium could be quite different in different states of the economy as we have noticed in our framework.

The other feature is that our framework highlights the dynamics of endogenous risks and these dynamics have substantial effects on the dynamics of bond-financing. In particular, as the banking sector becomes more and more financially healthy, endogenous risks becomes lower and lower, which in turn actually help firms issuing more corporate debt. This result implies the outstanding corporate debt in the economy is not monotonic in the state of the economy.

4 Bank Regulation: Quantity Control

Bank regulation could improve social welfare as it lowers the leverage of the banking sector directly and the leverage of the production sector indirectly. However, the negative effect of bank regulation is to prevent more productive sector raising external funds to fund their investments and thus to lower the aggregate productivity. In this section, we analyze welfare effects of the capital requirement through its impact on endogenous risks, the aggregate productivity, and the volatility and growth of different agents' wealth.

4.1 Capital Ratio Requirement

We consider the time-invariant capital ratio requirement, which imposes an upper bound on banks' loan-to-equity ratio, that is, $x_t \leq \bar{x}$.[6] While the constraint of capital ratio requirement is binding, the price of bank loans has to increase accordingly such that the demand for them declines. We can also observe the direct consequence of the capital ratio requirement based on bankers' first order condition, which is,

$$r_t^\lambda - r_t \geq \lambda(x_t^j + \lambda x_t)(\sigma + \sigma_t^q)^2, \text{with equality if } x_t < \bar{x}.$$

If the capital requirement constraint is binding, i.e., $x_t = \bar{x}$, then the positive Lagrange multiplier of the constraint implies that the loan r_t^λ is larger or equal to the level it would be if there were no such constraint. We next use results of model simulation to demonstrate how capital requirement constraint affects the aggregate economy.

In our simulation, we randomize initial states of 10,000 economies and simulate for 500 years.[7] In the end, we calculate the average values of endogenous variables that we are interested in.

[6]Recall that bankers are less productive than experts and households. Hence, x_t^j is positive in the equilibrium if and only if both experts and households are extremely poor and unable to hold all physical capital in the economy. In this case, households do not hold risk-free deposits issued by banks and it is unnecessary to discuss binding capital ratio requirement constraint.

[7]We randomize $\{\omega_i, i = 1, 2, ..., 10000\}$ according to the uniform distribution over $(0, 1)$ and generate η_i according to the uniform distribution over $(0, 1 - \omega_i)$ for each i.

4.2 Price and Quantity

Table 1 demonstrates the influence of capital ratio requirement on both price and quantity endogenous variables, such as net interest spread, the growth rate of physical capital, and the loan-to-bond ratio.

Rows 1 and 2 in Table 1 show that whereas tightening capital requirement increases the average *net interest spread*, its effects on the volatility of net interest spread are nonlinear. As the required capital ratio increases, banks' leverage declines (row 8 in Table 1). So does the supply of bank loans. Therefore, bank loans become more expensive and the average net interest spread widens. In addition, we notice that the volatility of the net interest spread substantially increases as the required capital rate rises from 0 to $1/13$. The underlying reason is that when the capital ratio requirement is modest, say around $1/15$, the capital constraint is not binding in normal periods when banks' leverage is relatively low. Hence, the net interest spread is low during normal periods. In recessions, however, the demand for bank loans is high and the capital requirement constraint binds. Therefore, the net interest spread rises significantly in recessions, which increases the volatility of the net interest spread. Nevertheless, this effect will be gone if the required capital ratio is so high (e.g., $1/6$) that banks' leverage restriction is always binding in the equilibrium.

Table 1: Impact of Quantity Control: Price and Quantity

Capital ratio $1/\bar{x}$	0	1/20	1/15	1/13	1/10	1/8	1/6	1/5	
Price									
$r_t^\lambda - r_t$	0.0258	0.0275	0.0326	0.0400	0.0595	0.0600	0.0600	0.0600	(1)
vol of $r_t^\lambda - r_t$ (%)	0.5092	1.0423	1.6499	1.8993	0.3645	0.0519	0.0412	0.0403	(2)
$q_t\sigma_t^q$	0.0790	0.0788	0.0756	0.0695	0.04987	0.0491	0.0489	0.0489	(3)
vol of $q_t\sigma_t^q$ (%)	1.8046	1.9294	2.1930	2.3359	0.8726	0.7566	0.7796	0.7880	(4)
Quantity on the financial side									
b_t^0	4.835	4.920	5.171	5.561	6.716	6.787	6.820	6.832	(5)
b^λ	0	0.018	0.199	0.575	2.153	2.438	2.570	2.624	(6)
risky firms' leverage [2]	4.2183	4.0628	3.7434	3.3879	2.6857	2.7468	2.7845	2.8007	(7)
external credit [3]	0.2922	0.2919	0.2852	0.2705	0.2252	0.2286	0.2304	0.2307	(8)
x_t	10.322	10.358	10.513	10.737	9.987	8	6	5	(9)
loan-to-bond ratio	3.3498	3.2480	2.8273	2.1516	0.1592	0.0622	0.0389	0.0310	(10)
Quantity on the real side									
μ_t^K (%)	0.00915	0.0997	0.1070	0.0850	-0.4088	-0.6152	-0.6806	-0.7054	(11)
TFP [4]	0.0613	0.0613	0.0608	0.0596	0.0532	0.0520	0.0517	0.0515	(12)

[1] We randomize initial states of 10,000 economies, stimulate them for 500 years, and report mean values of above endogenous variables except row 2 and 4, which show second order moments.

[2] Risky firms' leverage equals $b_t^\lambda + l_t$.

[3] "external credit" is the total external funds that firms raise as percentages of total wealth, which equals $(1-\alpha)\omega_t(b_t^\lambda + l_t) + \alpha b_t^0$.

[4] TFP is the average productivity of the entire economy.

Similar to the net interest spread, although tightening capital ratio requirement lowers the *price volatility of physical capital* on average, it has non-monotonic effects on the volatility of the price volatility physical capital (rows 3 and 4 in Table 1). More stringent capital requirement leads to lower endogenous risk because the leverage of both experts and bankers and their exposure to the exogenous risk decline (recall equation (25)). Modest capital requirement increases the volatility of the price volatility of physical capital for the following reason. Recall that the leverage of both experts and bankers is counter-cyclical. The capital requirement binds and loan rates rise exactly when the economy is in downturns, which substantially intensifies asset fire-sales and increases endogenous volatility in recessions. However, if the capital requirement is considerably tight, the constraint always binds in the equilibrium and the magnitude of asset fire-sales does not vary over business cycles. Neither does the volatility of the price volatility of physical capital.

Tighter capital ratio requirement leads to the lower total external credit that risky firms can raise. As banks are required to hold more equity capital, the supply of bank loans declines. Row 6 in Table 1 shows that risky firms gradually issue more corporate bonds when bank-finance becomes more expensive (row 1 in Table 1). However, we still observes the negative net effect of tightening capital requirement on risky firms' external financing given that the capital requirement is not tight enough (e.g., $\bar{x} \geq 10$, see row 7 in Table 1). The main reason is that bond-finance is much more expensive than bank-finance for risky firms. As they switch to the bond market, risky firms have to lower their leverage. The impact of raising capital requirement on the real economy is that risky firms have to lower their holdings of physical capital. Therefore, the average productivity tends to decline. So does the price of physical capital.

Although safe firms expands as capital requirement rises, stringent bank regulation has negative impacts on both total external credit the real sector raises and the average total-factor productivity. When the capital requirement increases, the leverage of risky firms that mainly rely on bank-finance declines. This lowers both the price of physical capital and its volatility. Hence, safe firms tends to expand their production (row 12 in Table 1) since the return of holding physical capital increases and the risk of holding physical capital declines. Nevertheless, the expansion of safe firms cannot overturn the result that the total external funds raise by firm declines substantially as capital requirement rises, say from $1/15$ to $1/10$ (row 8 in Table 1). The net effect of tightening capital requirement constraint on TFP is also the same. Even though safe firms hold more physical capital, the average TFP declines as capital ratio rises (row 12 in Table 1).

There is a striking result with respect to risky firms' external financing. If the required capital ratio is higher than $1/8$, tighter bank regulation actually induces risky firms to take higher leverage. The intuition of this result is that given the sufficiently tight bank regulation the endogenous risk is rather low and thus risky firms are able to take relatively high leverage. Hence, their overall leverage $b_t^\lambda + l_t$ is higher in an economy with a smaller supply of bank loans. As a consequence, when capital requirement is higher than $1/8$, more stringent regulation gives rise to more external funds that the real sector obtains from the credit market.

4.3 Experts, Bankers, and Households

Table 2 reports effects of tightening capital requirement on agents' welfare. In particular, we focus on the state where $\omega = 0.05$ and $\eta = 0.03$. Rows 1 and 3 in Table 2 show that as the capital ratio requirement increases from 0 to $1/10$, the welfare of both experts and households increases. However, if the capital requirement rises from $1/10$ to $1/8$ or higher, the welfare of the two types of agents deteriorates. With respect to the welfare of bankers, raising capital ratio requirement monotonically lowers their welfare. Tightening capital requirement affects agents' welfare through both the growth channel and the volatility channel, which I will discuss below in detail.

Tighter capital requirement generally leads to the lower volatility and growth of experts' wealth when the capital ratio requirement is below $1/10$ (rows 4 and 7 in Table 1). This is primarily because the constrained supply of bank loans lowers the leverage of risky experts, which in turn gives rise to their low exposure to aggregate risks and the low growth rate of their wealth. The welfare result indicates that the effect of low risk exposure dominates as the capital requirement is not tight enough (e.g., $\bar{x} \geq 10$).

If the capital ratio requirement is above $1/10$, more strict regulation makes the wealth of both households and experts more volatile. For experts, row 5-8 in Table 1 show that the leverage of both safe and risky experts increases as the required capital ratio rises from $1/10$ to $1/8$ or higher. Hence, experts have larger exposure to aggregate risks and thus their wealth becomes more volatile. To understand why tighter bank regulation creates more risks to households, we notice that *i* households are major bondholders and *ii* when risky firms default on their corporate bonds, it is bondholders who bear the aggregate risk. Hence, as risky firms issue more corporate bonds due to tight capital requirement, households' exposure to aggregate risk rises. When the capital ratio increases from $1/10$ to $1/5$, we observe that the welfare of both experts and households deteriorates as a result of high aggregate risks that they are exposed to.

Both the volatility and the growth rate of bankers' wealth decline due to the tightening of capital requirement (rows 5 and 8 in Table 1). However, the impact of low growth matters

Table 2: Impact of Quantity Control: Experts, Bankers, and Households

Capital ratio $1/\bar{x}$	0	1/20	1/15	1/13	1/10	1/8	1/6	1/5	
welfare									
expert	-211.04	-210.68	-210.32	-210.11	-210.5	-210.9	-211.6	-212.92	(1)
banker	-247.82	-248.2	-250.79	-255.38	-276.47	-296.78	-311.73	-316.04	(2)
household	-122.59	-122.56	-122.45	-122.41	-123.28	-124.45	-125.56	-126.49	(3)
wealth volatility									
expert	0.7168	0.7042	0.6740	0.6367	0.5617	0.5678	0.5709	0.5723	(4)
banker	0.5148	0.5142	0.5139	0.5101	0.4395	0.3526	0.2656	0.2222	(5)
household	0.1142	0.1138	0.1125	0.1104	0.1095	0.1110	0.1114	0.1116	(6)
wealth growth									
expert	0.2071	0.1976	0.1783	0.1577	0.1127	0.1141	0.1152	0.1158	(7)
banker	0.0483	0.0464	0.0444	0.0395	-0.0213	-0.0779	-0.1188	-0.1335	(8)
household	0.0083	0.0086	0.0079	0.0065	-0.0026	-0.0028	-0.0027	-0.0027	(9)

[1] All agents' welfare are evaluated at the state $\omega = 0.05$ and $\eta = 0.03$.
[2] We randomize initial states of 10,000 economies, stimulate them for 500 years, and report mean values of wealth volatility and wealth growth rate variables.

more as the welfare results shows (row 2 in Table 2). We also observe that although capital requirement constraint raises the interest spread that banks earn, their overall profitability decreases as the volume of loans that banks are able to originate declines. It is the quantity effect that lowers bankers' welfare. This effect, however, does not apply to experts because given the limited supply of bank loans experts can still resort to bond-financing.[8]

5 Bond Market and Bank Regulation

This section shows that the analysis of optimal required capital ratio can be misleading if we only consider the relative importance of bond-finance without fully taking into account the fundamental reason why an economy replies more on bond-finance. In particular, we explore three possible reasons why bond-finance could be more prominent in an economy and characterize the plausible optimal capital requirement. First, bond-finance is more popular in economies with better developed bond markets, that is, the costly liquidation by bondholders is more efficient in such economies (small κ^d). Second, there will be more bond-finance if more safe firms are operating in economies (large α). Third, how risky those risky firms are (i.e., the level of λ) also matters for the volume of bond-finance in an economy.

In this section, we only focus on bank regulation's welfare implications for experts and households as Section 4 shows that imposing capital ratio requirement always lowers bankers'

[8] Recall that experts are identical in the beginning of each period.

welfare. It is beyond the scope of our paper to discuss how the society should weigh bankers' welfare against the rest of the economy.

5.1 More Efficient Bond Market

In this section, we highlight two observations of our comparative statics analysis. First, tightening capital requirement may actually lower financial stability for an economy with a more advanced market. Second, the optimal capital requirement should be more lenient in this economy.

The development of bond market lowers the cost of firm liquidation that bondholders initiate, that is, to lower κ^d. In this section, we take the numerical example in Section 3 as the benchmark and vary parameter κ^d to investigate the implication of bond markets for bank regulation. In particular, we consider two alternative economies where $\kappa^d = 0.3$ and $\kappa^d = 0.6$ with all other parameters having the same values as the benchmark case.

Table 3: More Efficient Bond Market: welfare implication of bank regulation

This table reports endogenous variables of three economies: the benchmark ($\kappa = 0.4$), one with less developed bond market ($\kappa = 0.3$), and one with more developed bond market ($\kappa = 0.6$). Columns "5" - "∞" report the difference of reported variables between two economies, one with required capital ratio 1/5 and one without capital requirement. All agents' welfare are evaluated at the state $\omega = 0.05$ and $\eta = 0.03$. We randomize initial states of 10,000 economies, stimulate them for 500 years, and report mean values of wealth volatility and wealth growth rate variables as well as other endogenous price and quantity variables.

	$\kappa^d = 0.3$				$\kappa^d = 0.4$ (benchmark)			$\kappa^d = 0.6$			
	$\bar{x} = \infty$	$\bar{x} = 10$	$\bar{x} = 5$	"5"-"∞"	$\bar{x} = \infty$	$\bar{x} = 5$	"5"-"∞"	$\bar{x} = \infty$	$\bar{x} = 10$	$\bar{x} = 5$	"5"-"∞"
price and quantity											
$r_t^\lambda - r_t$	0.026	0.038	0.030	0.004	0.060	0.034	0.026	0.026	0.086	0.088	0.062
$q_t\sigma_t^q$	0.079	0.065	0.075	-0.004	0.079	0.049	-0.030	0.079	0.040	0.037	-0.042
b^0	4.842	5.885	4.981	0.139	4.835	6.832	1.997	4.819	7.820	8.093	3.274
b^λ	0.764	2.678	3.849	3.085	0	2.624	2.624	0	0	0	0
risky firms' leverage	4.219	3.384	4.039	-0.180	4.218	2.801	-1.418	4.207	0.247	0.060	-4.148
external credit	0.293	0.284	0.283	-0.010	0.292	0.231	-0.061	0.291	0.202	0.195	-0.096
loan-to-bond ratio	2.410	0.259	0.027	-2.383	3.350	0.031	-3.319	3.353	0.151	0.028	-3.324
μ_t^K (%)	0.042	0.297	-0.436	-0.477	0.091	-0.705	-0.796	0.080	0.608	0.683	0.603
TFP	0.062	0.064	0.061	-0.001	0.061	0.052	-0.009	0.061	0.052	0.051	-0.010
welfare											
expert	-211	-206.6	-213.1	-2.13	-211.0	-212.9	-1.88	-211.3	-201.4	-190.4	20.82
banker	-251.66	-268.2	-312.2	-60.55	-247.8	-316.0	-68.2	-248.04	-275.8	-308.38	-60.34
household	-122.76	-121.3	-124.81	-2.05	-122.6	-126.5	-3.9	-122.68	-121.5	-118.68	4
wealth volatility											
expert	0.717	0.645	0.706	-0.011	0.717	0.572	-0.144	0.717	0.367	0.353	-0.364
banker	0.516	0.474	0.259	-0.258	0.515	0.222	-0.293	0.515	0.407	0.200	-0.315
household	0.115	0.111	0.121	0.006	0.114	0.112	-0.002	0.115	0.104	0.103	-0.012
wealth growth											
expert	0.208	0.114	0.196	-0.012	0.207	0.116	-0.091	0.207	0.032	0.029	-0.178
banker	0.042	-0.056	-0.128	-0.170	0.048	-0.134	-0.182	0.048	-0.039	-0.136	-0.184
household	0.008	-0.051	0.002	-0.006	0.008	-0.003	-0.01	0.008	0.011	0.012	0.004

Economies with less developed bond market tend to have higher loan-to-bond ratios as Table 3 indicates. This is intuitive because the higher liquidation cost incurred by bond-finance leads to more costly bond-finance, which makes risky firms prefer bank-finance.

Our numerical exercise shows that the optimal capital requirement (from perspectives of experts and households) should be more lenient in an economy with a more advanced bond market. Table 3 and Figure 10 show that the welfare of both experts and households declines as the required capital ratio increases from $1/10$ to $1/5$ in the economy with more developed bond market ($\kappa^d = 0.3$). However, the same regulatory change improves the two groups of agents' welfare in a less mature bond market ($\kappa^d = 0.6$). Next, we will outline the main intuition underlying this result.

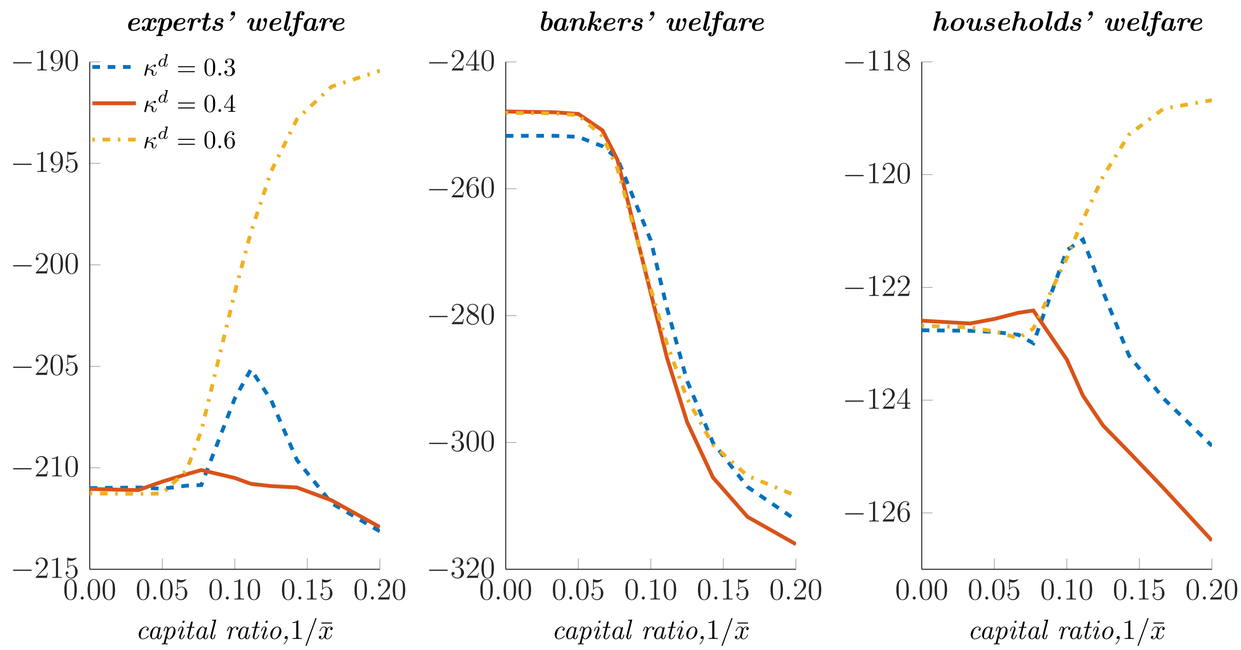

Figure 10: More efficient bond market: welfare comparison of bank regulation
This figure displays how the welfare of experts (the left panel), bankers (the middle panel), and households (the right panel) change as the required capital ratio varies in three different economies. The blue dashed lines represent economies with $\kappa^d = 0.3$, the red solid lines economies with $\kappa^d = 0.4$, and the orange dot-dashed lines economies with $\kappa^d = 0.6$.

In an economy with a better bond market, tightening capital requirement cannot effectively lower the leverage of the real sector. Risky firms find it less costly to replace bank loans with corporate bonds if the bond market is more efficient. Therefore, tightening capital requirement does not significantly change either the overall leverage of risky firms or the total external credit that the real sector as Table 3 shows. In particular, as the required capital ratio rises from $1/10$ to $1/5$, the overall leverage of risky firms and the total external funds that the real sector raises go up in the economy with an efficient bond market ($\kappa^d = 0.3$). This is the opposite of what occur in the economy with the less efficient bond market ($\kappa^d = 0.6$).

The reason why the leverage of risky firms can increase given the strengthening of capital requirement is that the price of physical capital declines in the presence of tight regulatory restriction. And, the decline in the price of physical capital raises the return of holding physical capital. Thus, risky firms have stronger incentives to take higher leverage.

In the economy with a more advanced bond market, tightening capital requirement effects financial stability in a counter-productive way. Table 3 shows that as the capital requirement rises from $1/10$ to $1/5$, endogenous risk increases in the economy with a more efficient market ($\kappa^d = 0.3$) and declines in the economy with a less efficient market ($\kappa^d = 0.6$). The intuition is that in an economy where bond-finance is more costly, risky firms cannot easily switch to bond-finance and thus bank regulation is more effective in terms of lowering risky firms' leverage. This, however, does not work in an economy with a more efficient bond market. What is worse is that the often switch between bank credit and bond credit increases the risk of asset fire-sale as well as endogenous risk.

Similar to endogenous risk, as the capital requirement rises from $1/10$ to $1/5$, the wealth volatility of both experts and households also increases in the economy with a more advanced bond market ($\kappa^d = 0.3$). As the two types of agents' wealth becomes more volatile, their welfare deteriorates. This explains why the optimal bank regulation should be more lenient when the bond market is more efficient in an economy.

5.2 Less Risky Firms

The relatively low loan-to-bond ratio could also be the consequence of the fact that there are less risky firms in an economy. In this section, we vary the fraction of safe experts (i.e., α) in an economy while keeping all other parameter values unchanged.

Table 4 shows that in an economy with more safe experts/firms the loan-to-bond ratio is lower and the average TFP and the growth rate of physical capital are higher. These results are intuitive. Since safe firms do not rely on bank-finance at all, the loan-to-bond ratio is lower in economies with less safe firms. Safe firms are able to take higher leverage than risky ones because safe firms do not pay any premium for the cost of firm liquidation. Hence, the average TFP and the growth rate of physical capital are higher when there are more safe firms.

As Figure 11 shows, the optimal capital requirement is more lenient in an economy with more risky firms that mainly rely bank-finance. In particular, raising capital ratio from$1/10$ to $1/5$ uniformly lowers the welfare of all three types of agents in the economy with only 10% of firms being safe. This, however, is untrue for the economy with safe firms taking 30% of

Table 4: Less Risky Firms: welfare implication of bank regulation

This table reports endogenous variables of three economies: the benchmark ($\alpha = 0.2$), one with less safe firms ($\alpha = 0.1$), and one with more safe firms ($\alpha = 0.3$). Columns "5" - "∞" report the difference of reported variables between two economies, one with required capital ratio 1/5 and one without capital requirement. All agents' welfare are evaluated at the state $\omega = 0.05$ and $\eta = 0.03$. We randomize initial states of 10,000 economies, stimulate them for 500 years, and report mean values of wealth volatility and wealth growth rate variables as well as other endogenous price and quantity variables.

	$1-\alpha = 0.9$				$1-\alpha = 0.8$ (benchmark)			$1-\alpha = 0.7$			
	$\bar{x} = \infty$	$\bar{x} = 10$	$\bar{x} = 5$	"5"-"∞"	$\bar{x} = \infty$	$\bar{x} = 5$	"5"-"∞"	$\bar{x} = \infty$	$\bar{x} = 10$	$\bar{x} = 5$	"5"-"∞"
price and quantity											
$r_t^\lambda - r_t$	0.025	0.060	0.060	0.035	0.026	0.060	0.034	0.027	0.058	0.060	0.033
$q_t \sigma_t^q$	0.075	0.042	0.041	-0.034	0.079	0.049	-0.030	0.019	0.011	0.008	-0.011
b^0	5.083	7.689	7.820	2.737	4.835	6.832	1.997	4.593	5.825	5.961	1.368
b^λ	0	2.950	3.658	3.658	0	2.624	2.624	0	1.521	1.904	1.904
risky firms' leverage	4.572	3.656	3.903	-0.670	4.218	2.801	-1.418	3.880	2.069	2.055	-1.825
external credit	0.281	0.179	0.186	-0.096	0.292	0.231	-0.061	0.302	0.264	0.267	-0.035
loan-to-bond ratio	7.815	0.235	0.042	-7.773	3.350	0.031	-3.319	1.880	0.164	0.026	-1.854
μ_t^K (%)	-0.306	-1.496	-1.925	-1.619	0.091	-0.705	-0.796	0.490	0.600	0.442	-0.048
TFP	0.058	0.042	0.040	-0.018	0.061	0.052	-0.009	0.064	0.062	0.061	-0.003
welfare											
expert	-218.6	-224.6	-236.4	-17.84	-211.0	-212.9	-1.88	-203.8	-199.6	-195.5	8.31
banker	-249.4	-283.6	-327.8	-78.34	-247.8	-316.0	-68.2	-247.0	-270.1	-305.6	-58.57
household	-126.8	-130.6	-138.9	-12.05	-122.6	-126.5	-3.9	-118.4	-117.0	-116.1	2.33
wealth volatility											
expert	0.715	0.571	0.592	-0.123	0.717	0.572	-0.144	0.721	0.574	0.575	-0.146
banker	0.509	0.424	0.215	-0.294	0.515	0.222	-0.293	0.520	0.458	0.230	-0.290
household	0.116	0.114	0.116	0.001	0.114	0.112	-0.002	0.113	0.106	0.107	-0.006
wealth growth											
expert	0.200	0.107	0.113	-0.087	0.207	0.116	-0.091	0.215	0.128	0.129	-0.087
banker	0.043	-0.032	-0.136	-0.179	0.048	-0.134	-0.182	0.053	-0.009	-0.130	-0.184
household	0.004	-0.015	-0.016	-0.020	0.008	-0.003	-0.01	0.013	0.009	0.010	-0.003

the population. At least, raising capital ratio from 1/10 to 1/5 increase the welfare of both experts and households in the latter economy.

It is straightforward to interpret the above welfare implication. Implementing capital requirement considerably lowers the leverage of risky firms. Given that risky firms are the majority of the production sector, capital ratio requirement causes more substantial declines in average TFP and the growth in physical capital in economies with more risky firms (Table 4). Although risky firms can substitute increasingly expensive bank credit with bond credit as capital requirement tightens, the total external credit that the real sector can raise still declines substantially. Therefore, we see that the optimal capital requirement should be more lenient in an economy with more risky firms.

By combining results in this section and Section 5.1, we observe that it is not always true that the optimal capital requirement ought to be more lenient in an economy with a relatively high loan-to-bond ratio. The exercise in this section shows that economies with less safe firms should have relatively lenient capital ratio requirement than those with more

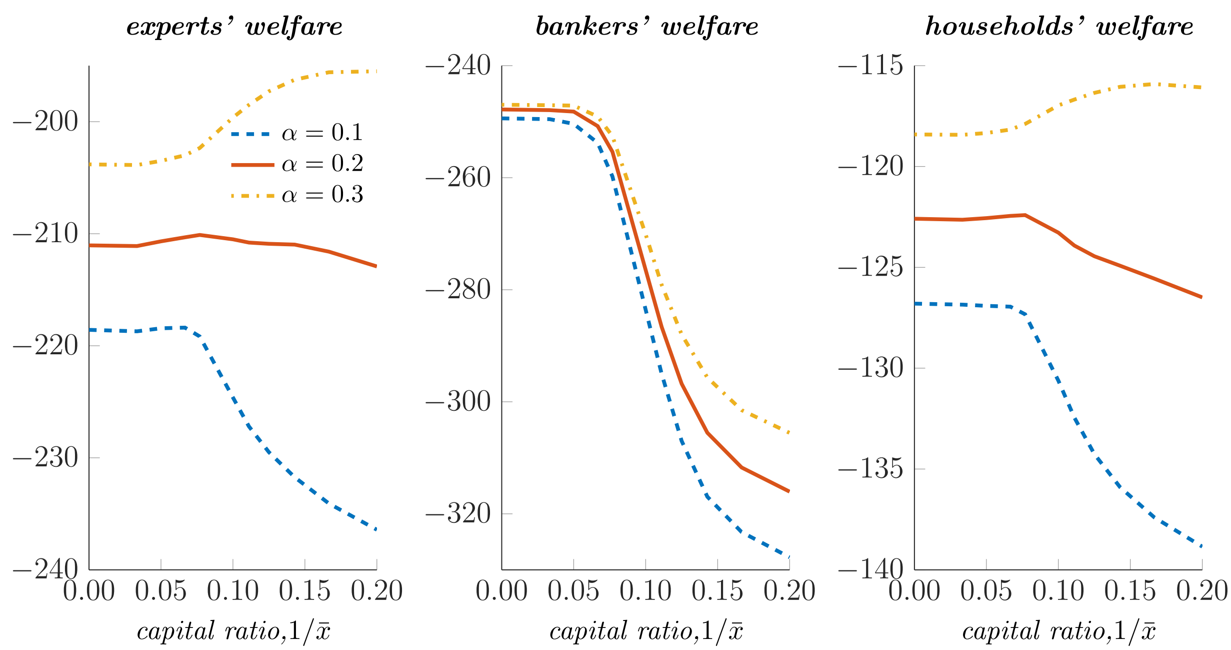

Figure 11: Less risky firms: welfare implication of bank regulation
This figure displays how the welfare of experts (the left panel), bankers (the middle panel), and households (the right panel) change as the required capital ratio varies in three different economies. The blue dashed lines represent economies with $\alpha = 0.1$, the red solid lines economies with $\alpha = 0.2$, and the orange dot-dashed lines economies with $\alpha = 0.3$.

safe firms. Since safe firms do not rely on bank-finance, economies with less safe firms have relatively high loan-to-bond ratios. However, in Section 5.1 it is economies with relatively low loan-to-bond ratios that should have less strict capital requirement (Table 3). Overall, we show that it could be misleading if we discuss the optimal capital ratio without examining the exact reason why bank-finance is relatively prominent in an economy.

5.3 Risky Firms Are Riskier

The overall risk of all firms in an economy not only depends on the fraction of safe firms but also relies on idiosyncratic risks of risky firms. In this section, we examine the regulatory implications of changes in the magnitude of idiosyncratic risk of risky firms (i.e., λ).

The first interesting we observe is that if there is no bank regulation, the loan-to-bond ratio is higher in the economy whose risky firms are safer; and if capital requirement is sufficiently tight (e.g., $\bar{x} = 10$), the loan-to-bond ratio is higher in the economy whose risky firms are riskier. The funding cost of bank-finance is relatively lower for risky firms that are safer. Hence, these firms tend to choose more bank-finance when the supply of bank loans is not regulated. However, when capital requirement is in place, the supply of bank loans diminishes and bank loans become more expensive. Risky firms that are relatively safer find it easier to switch to bond credit. Therefore, the loan-to-bond ratio is lower in an economy where risky firms are less risky when capital requirement is in place.

Table 5: Risky Firms are Risker: welfare implication of bank regulation

This table reports endogenous variables of three economies: the benchmark ($\lambda = 0.3$), one with risky firms being relatively safer ($\lambda = 0.25$), and one with risky firms being relatively riskier ($\lambda = 0.35$). Columns "5" - "∞" report the difference of reported variables between two economies, one with required capital ratio 1/5 and one without capital requirement. All agents' welfare are evaluated at the state $\omega = 0.05$ and $\eta = 0.03$. We randomize initial states of 10,000 economies, stimulate them for 500 years, and report mean values of wealth volatility and wealth growth rate variables as well as other endogenous price and quantity variables.

	$\lambda = 0.25$				$\lambda = 0.3$ (benchmark)			$\lambda = 0.35$			
	$\bar{x} = \infty$	$\bar{x} = 10$	$\bar{x} = 5$	"5"-"∞"	$\bar{x} = \infty$	$\bar{x} = 5$	"5"-"∞"	$\bar{x} = \infty$	$\bar{x} = 10$	$\bar{x} = 5$	"5"-"∞"
price and quantity											
$r_t^\lambda - r_t$	0.022	0.050	0.050	0.028	0.026	0.060	0.034	0.029	0.061	0.068	0.040
$q_t\sigma_t^q$	0.086	0.063	0.063	-0.023	0.079	0.049	-0.030	0.072	0.044	0.035	-0.037
b^0	4.473	5.723	5.779	1.305	4.835	6.832	1.997	5.257	7.468	8.199	2.942
b^λ	7×10^6	2.897	3.216	3.216	0	2.624	2.624	0	0.730	1.085	1.085
risky firms' leverage	4.057	3.336	3.404	-0.653	4.218	2.801	-1.418	4.343	1.723	1.193	-3.150
external credit	0.303	0.246	0.250	-0.054	0.292	0.231	-0.061	0.280	0.234	0.222	-0.058
loan-to-bond ratio	3.499	0.089	0.030	-3.469	3.350	0.031	-3.319	3.158	0.643	0.0278	-3.130
μ_t^K (%)	0.476	-0.529	-0.674	-1.149	0.091	-0.705	-0.796	-0.265	0.192	0.283	0.548
TFP	0.064	0.055	0.054	-0.010	0.061	0.052	-0.009	0.058	0.0555	0.054	-0.004
welfare											
expert	-204.1	-208.9	-214.6	-10.42	-211.0	-212.9	-1.88	-217.8	-211.8	-197.4	20.34
banker	-246.8	-288.7	-317.5	-70.7	-247.8	-316.0	-68.2	-249.8	-266.6	-307.8	-57.94
household	-118.5	-122.0	-126.8	-8.31	-122.6	-126.5	-3.9	-126.5	-124.9	-120.0	6.56
wealth volatility											
expert	0.738	0.649	0.655	-0.083	0.717	0.572	-0.144	0.691	0.473	0.431	-0.260
banker	0.519	0.398	0.202	-0.317	0.515	0.222	-0.293	0.510	0.467	0.232	-0.278
household	0.114	0.116	0.118	0.003	0.114	0.112	-0.002	0.114	0.102	0.099	-0.014
wealth growth											
expert	0.226	0.160	0.163	-0.063	0.207	0.116	-0.091	0.186	0.075	0.058	-0.128
banker	0.050	-0.056	-0.142	-0.192	0.048	-0.134	-0.182	0.046	0.006	-0.126	-0.171
household	0.013	-0.001	-0.001	-0.014	0.008	-0.003	-0.01	0.004	0.005	0.006	0.002

The welfare implication of this numerical exercise is similar to the one in Section 5.1. Basically, the optimal capital requirement should be more lenient in an economy where risky firms can relatively easily substitute bank credit with bond credit. The intuition is also similar to that in Section 5.1. As the required capital ratio increases, risky firms that are relatively less risky (low λ) find it less expensive to replace bank-finance with bond-finance. Therefore, the overall leverage of these risky firms does not decrease as significantly as it does in an economy where risky firms find it more costly to replace bank credit because they are relatively riskier.

Results in this section also imply we cannot only use the loan-to-bond ratio to judge whether it is optimal to have more lenient or more stringent regulation. As Table 5 and Figure 12 show, if the capital requirement is not implemented, it is the economy with a high

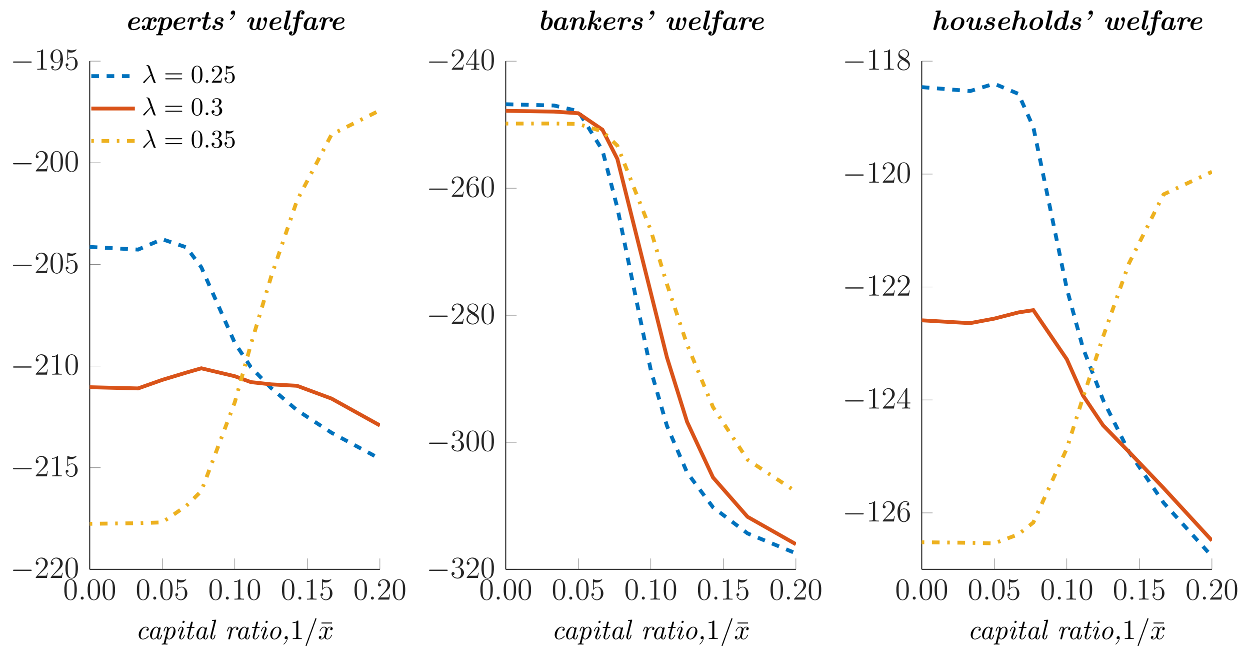

Figure 12: Risky firms are riskier: welfare comparison of bank regulation
This figure displays how the welfare of experts (the left panel), bankers (the middle panel), and households (the right panel) change as the required capital ratio varies in three different economies. The blue dashed lines represent economies with $\lambda = 0.25$, the red solid lines economies with $\lambda = 0.3$, and the orange dot-dashed lines economies withe $\lambda = 0.35$.

loan-to-bond ratio that should have looser capital requirement; if the capital requirement (e.g., $\bar{x} = 10$) is already in place, then it is the economy with a low loan-to-bond ratio that deserves more lenient regulation. Therefore, what really matters for capital requirement is the fundamental reason why bond-finance is relatively more prominent in an economy.

6 Conclusion

In this paper, we present a dynamic general framework, in which firms can access both bank credit and bond credit, and banks channel credit from savers to borrowers. The intermediation cost of bank-financing fluctuates endogenously because the risk-premium that banks ask for depends on the financial health of the banking sector. We investigate aggregate impacts of capital ratio requirement in our framework. For bank regulation, we particularly focus on capital ratio requirement as quantity control.

We find that the analysis of optimal capital requirement should not just focus an economy's reliance on bond-finance but also its underlying micro-foundation. We show that capital requirement ought to be lenient for economies with relatively less safe firms that easily access the bond market. And, for economies where risky firms can easily switch between bank-finance and bond-finance, the optimal capital ratio should also be relatively low.

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Appendix

A Proofs

Proof of Lemma 1.

To apply Ito's Lemma, we first have

$$\mathrm{d}\big(q_t K_t\big) = q_t K_t(\mu_t^q + \mu_t^K + \sigma\sigma_t^q)\mathrm{d}t + q_t K_t(\sigma + \sigma_t^q)\mathrm{d}Z_t.$$

Given the above equation, equation 19, and Ito's Lemma, we have

$$\begin{aligned}
\mathrm{d}\omega_t = & \frac{W_t}{q_t K_t}\Big(R_t + \alpha b_t^0(R_t - r_t) + (1-\alpha)b_t^\lambda(R_t + \delta^r - \lambda\kappa^d - r_t) + (1-\alpha)l_t(R_t + \delta^r - \lambda\kappa - r_t^\lambda) - \frac{c_t}{W_t}\Big)\mathrm{d}t \\
& - \frac{W_t}{q_t K_t}(\mu_t^q + \mu_t^K + \sigma\sigma_t^q)\mathrm{d}t - \frac{W_t}{q_t K_t}\Big(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)(1-\lambda)\Big)(\sigma + \sigma_t^q)^2\mathrm{d}t \\
& + \frac{W_t}{q_t K_t}(\sigma + \sigma_t^q)^2\mathrm{d}t + \frac{W_t}{q_t K_t}\Big(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)(1-\lambda)\Big)(\sigma + \sigma_t^q)\mathrm{d}Z_t - \frac{W_t}{q_t K_t}(\sigma + \sigma_t^q)\mathrm{d}Z_t \\
\frac{\mathrm{d}\omega_t}{\omega_t} = & \mu_t^\omega \mathrm{d}t + \sigma_t^\omega \mathrm{d}Z_t.
\end{aligned}$$

Given bankers' Euler equation (15), the law of motion for W_t can be rewritten as

$$\frac{\mathrm{d}N_t}{N_t} = \Big((x_t^j + \lambda x_t)^2(\sigma + \sigma_t^q)^2 + r_t - \frac{c_t}{N_t}\Big)\mathrm{d}t + (x_t^j + \lambda x_t)(\sigma + \sigma_t^q)\mathrm{d}Z_t.$$

Hence,

$$\begin{aligned}
\mathrm{d}\eta_t = & \frac{N_t}{q_t K_t}\Big((x_t^j + \lambda x_t)^2(\sigma + \sigma_t^q)^2 + r_t - \frac{c_t}{N_t}\Big)\mathrm{d}t - \frac{N_t}{q_t K_t}(\mu_t^q + \mu_t^K + \sigma\sigma_t^q)\mathrm{d}t \\
& - \frac{N_t}{q_t K_t}(x_t^j + \lambda x_t)(\sigma + \sigma_t^q)^2\mathrm{d}t + \frac{N_t}{q_t K_t}(\sigma + \sigma_t^q)^2\mathrm{d}t + \frac{N_t}{q_t K_t}(x_t^j + \lambda x_t)(\sigma + \sigma_t^q)\mathrm{d}Z_t - \frac{N_t}{q_t K_t}(\sigma + \sigma_t^q)\mathrm{d}Z_t \\
\frac{\mathrm{d}\eta_t}{\eta_t} = & \mu_t^\eta \mathrm{d}t + \sigma_t^\eta \mathrm{d}Z_t.
\end{aligned}$$

■